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ARTICLE

# Economically Optimal Soil Sampling Density and Application Technology for Potassium Fertilizer for Soybean in the Mid-Southern US

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## ABSTRACT

To maximize beneficial potassium (K) fertilizer use in irrigated soybean (*Glycine max.* (L.) Merr.) fields with spatially varying soil-test K (STK), the value of added information from more precise STK maps must be greater than the associated information collection costs. In eleven fields, we modeled the impact of soil sampling densities (SD) ranging from 2.2 samples ha-1 in the largest field (41.2 ha) to 13.59 samples ha-1 in the smallest field (7.4 ha) on STK maps with 0.4 ha grid size. The accuracy of profit-maximizing fertilizer rate prescription maps varied by SD and subsequent yield estimates using either uniform rate technology (URT) or variable rate technology (VRT). Fertilizer rate recommendations also depended on: i) the expected field yield; ii) the crop price; and iii) the fertilizer cost, costs for fertilizer application, and information collection charges that varied by application technology. Relative profitability comparisons across SD and fields revealed that collecting more than 1.1 samples ha-1 was not viable. URT was more profitable than VRT (ranging from \$2.29 ha-1 to \$7.62 ha-1) at both relatively low and high field-level average STK and spatial variation in STK. At the mid-range level of STK, where adding K-fertilizer was on the

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verge of being profitable in light of nearly adequate STK, VRT outperformed URT in two of eleven fields by $11.50 to $21.35 ha-1. Regardless of soybean price and fertilizer cost, a smaller upcharge for VRT compared to URT fertilizer application than the $5 ha-1 modeled herein is necessary to increase VRT viability.

**Highlights**

- Across eleven fields soil-test K (STK) spatial variance increased as field average STK increased.
- At high STK (> 130 mg K kg-1), fertilizer application was not profitable in the short term.
- None of the fields supported a soil sampling density greater than 1.1 samples ha-1.
- Greater STK map accuracy with more soil samples led to mixed revenue impacts when K fertilizer was needed with low STK.
- VRT was rarely more profitable than URT except in fields with mid-range average and standard deviation of STK.

***Keywords:*** Soybean; Potassium Fertilizer; Soil Sampling Density; Variable Rate Technology

## 1. Introduction

To maximize beneficial potassium (K) fertilizer use in soybean (*Glycine max.* (L.) Merr.) fields with spatially varying soil-test K (STK), the value of added information from greater spatial detail in STK maps must be greater than associated information collection costs. This is especially true given three motivating factors. First, the cost of commercial fertilizer has shown some recent price peaks. In 2022, the average cost of muriate of potash or K fertilizer reached a 13-year peak of \$0.863 kg$^{-1}$ K. Prices have since moderated, but projections are subject to change given persistent challenges faced by major global fertilizer suppliers [1]. Importantly, changes in K cost affect profit-maximizing K-fertilizer rates in agricultural production [2, 3].

Second, the cost of soil sampling and field fertilizer input prescription mapping services has increased noticeably. These information collection costs need to be justified by the benefits associated with the adoption of precision agriculture (PA) technology by producers who employ variable rate fertilizer application (VRT). Spatial soil sampling recommendations by agronomists and agricultural extension specialists are influenced by substantial year-to-year fluctuations in spatial soil nutrient levels. These fluctuations are driven by climate variability, such as changes in rainfall patterns, temperature variations, and extreme weather events, which affect crop nutrient uptake, soil moisture, and microbial activity. Moreover, high input costs and budget constraints often lead to reduced fertilizer applications, further contributing to nutrient imbalances and spatial variability [4].

Finally, the benefit of using VRT versus Uniform Rate Technology (URT) is nuanced. Analyzing soil nutrient data from multiple fields provides some insights into the reasons behind the complexity of fertilizer rate recommendations. For instance, Späti et al. [5] showed that high-resolution sensing increases profits in fields with high spatial heterogeneity, though overall benefits and differences between technologies remain modest. Despite considerable experimental research, reaching a definite conclusion on soil sampling density and fertilizer application rates is hindered by the variability of environmental factors from year to year [6, 7]. Conversely, some studies provide evidence supporting VRT as the most economically desirable technology. However, certain expense variables, such as the cost of soil sampling, mapping, and additional upcharges for technology, are omitted from those analyses, which complicates the interpretation of economic conclusions drawn from those efforts [8, 9].

Despite the concept of VRT gaining significant interest since the early 1990s in North America, the adoption rate among producers is still low, ranging between 20 and 30 percent, suggesting that producers are still not quite convinced about its profitability [10]. The concept of an Economic Optimum Sampling Density (EOSD) mentioned by Lawrence [11] suggests more work is needed to identify what sampling density maximizes field profitability for either URT or VRT fertilizer application. The field's EOSD entails comprehensive investigation of both agronomic (level and spatial variability of existing soil nutrient content and crop grown) and economic factors, including the cost per soil sample, the price of fertilizer, crop yield potential, the crop's price, and differential application cost between URT vs. VRT.

Challenges arise in optimizing the "4Rs" (Right source, Right rate, Right time, Right location) of soil nutrient application due to significant in-field variation in soil nutrient levels and extreme weather impacts [12]. The expected marginal value product of the additional information (from greater soil sampling density) increases at a diminishing rate [13], which is supported by a recent study involving the simulation of irrigated soybean yield response to fertilizer K under varying soil sampling densities [14]. Badarch et al. [14] analyzed the tradeoff of better nutrient matching between plant needs and available soil K reserves using VRT fertilizer application that varied by grid vs. using the same URT fertilizer application rate across a field. At issue was how the estimated average STK value in the field varied when changing soil sampling density, and more importantly, how the spatial distribution of STK maps changed with soil sampling density. They concluded that URT performs better than VRT, given the soil sampling cost and upcharges for VRT fertilizer application in comparison to URT fertilizer application. The analysis was limited to one field, however. Hence, this research expands on Badarch et al.'s [14] work by replicating their approach across more fields to allow greater generalization of findings. For example, Murdock and Howe [15] found that larger fields exhibit greater spatial variability in STK, justifying the use of VRT. At the same time, VRT for phosphorus (P) and potassium (K) fertilization could be profitable, especially when a field had a mix of high and low soil test levels, with at least 50% testing high. However, VRT is generally not cost-effective when most of the field tests are in the high to medium range.

Given this background information, this research is framed around the following three main questions. First, does added soil sampling in a field result in sufficient extra soybean yield or K fertilizer cost savings to warrant investment, and if so, at what sampling density? Second, does the answer to the first question vary by field? Third, is there a rule of thumb for when VRT is more profitable than URT based on average and spatial variance of STK, crop price, fertilizer cost, and sampling density? Using the average and the standard deviation of STK in eleven fields, we attempt to predict profitability differences between VRT and URT in this research. We explore the relationship between the standard deviation of STK, the average STK, the field size, and the soil sampling density to assist with finding a heuristic that would guide producers as they make choices related to soil sampling and what fertilizer application method to pursue.

Specifically, we analyzed STK data from eleven irrigated soybean fields exhibiting different initial STK values and spatial STK distributions. Each field's soybean yields were estimated using profit-maximizing fertilizer K rates ($K^*$) using a yield response function to K fertilizer developed by Popp et al. [3] which is subject to i) STK in each grid for VRT and the overall average field STK for URT; ii) crop price; and iii) fertilizer cost. Using $K^*$, we obtain field partial returns as a function of simulated yield by grid, crop price, fertilizer costs, soil sampling and fertilizer application charges that vary by application method (URT or VRT). Like Badarch et al. [14] we analyze the tradeoff of cost savings from lesser sampling to net losses that result with nutrient mismatch between plant needs and available K because of lesser STK map accuracy in attendant fertilizer prescription maps that impact yields and fertilizer use. We then compare profitability differences between URT and VRT at various sampling densities and across all fields in search of factors that determine what field characteristics impact those profitability differences.

## 2. Materials and Methods

### 2.1. Data and Mapping

The STK data used in this research were gathered from eleven farm fields across three different regions in Arkansas. Four fields are in St. Francis County in East Central Arkansas, four are in Drew and Lincoln counties in Southeastern Arkansas, and three are in Conway County in Central Arkansas (**Figure 1**). All STK samples were collected to a soil depth of 0–10 cm in the spring season of 2022. Each field's STK statistics are summarized in **Table 1** and vary based on sampling density. The overall average of all fields' STK is for relative comparisons across fields. Soil sampling density ranged from 2.23 to 13.59 samples $ha^{-1}$ (top rows for each field) and varied by field size. County and state average STK information is provided in **Table 2** to compare the eleven fields analyzed to centralized state-level soil testing information.

To measure the impact of reduced sampling density, each field's initial maximum number of soil samples (multiplying the size of the field times the sampling density in the top row of each field in **Table 1**) was successively cut in half four times to showcase how reducing soil sampling density impacts field STK statistics. Since most prescription mapping software uses the Inverse Distance Weighting (IDW) interpolation method to interpolate from soil sample locations to grids [17], all STK maps were generated in this manner using ArcGIS Pro software (ESRI, Redlands, California, USA). Further, all STK maps utilized a 20 m x 20 m (400 $m^2$) fishnet grid size for two main reasons: 1) application equipment width, potentially with section control, is expected to allow different rates in 20 m wide paths, 2) application speeds of up to 4.5 m $s^{-1}$ and anticipatory rate change time requirements of 2 s suggest that application rate changes every 20 m are possible. In line with Badarch et al. [14], we assume fertilizer rate changes between grids occur in 5.6 kg K $ha^{-1}$ increments. For illustration purposes, the five different soil sampling densities for STK maps in field 1 are presented in **Figure 2.** Notice that grids only partially contained near the fields' boundary were ignored, as was the selection of an optimal field path based on field irregularities.

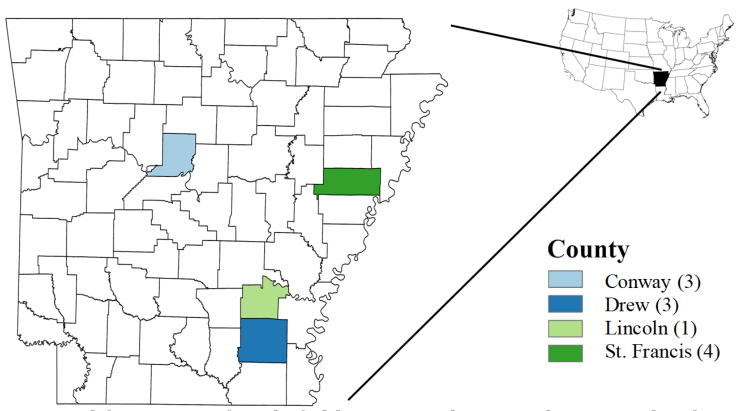



**Figure 1.** Locations and frequency of study fields in parentheses at the county level in Arkansas, USA.

**Table 1.** Descriptive statistics of Mehlich-3 extractable soil-test K (STK) to 10 cm soil depth across eleven fields that varied by location, size, soil sampling density, and spatial variation in STK, Arkansas, 2022.

| County | Size (i) | Sampling Density[1] (j) | ID (k) | Field STK Map Statistics per Field in mg K $kg^{-1}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Average | Median | Standard Deviation | CV(%) | Min. | Max. |
| Lincoln | 41.2 | 2.23 | 1 | 204.6 | 196.1 | 73.5 | 35.9 | 79.4 | 377.8 |
| | | 1.09 | | 204.1 | 200.3 | 68.5 | 33.5 | 84.2 | 366.2 |
| | | 0.53 | | 211.8 | 192.3 | 65.8 | 31.1 | 79.5 | 333.1 |
| | | 0.27 | | 227.1 | 211.6 | 66.1 | 29.1 | 78.3 | 374.3 |
| | | 0.12 | | 218.2 | 187.9 | 56.2 | 25.8 | 138.1 | 311.8 |
| Drew | 14.3 | 6.99 | 2 | 198.1 | 171.1 | 68.3 | 34.5 | 120.1 | 416.2 |
| | | 3.50 | | 203.2 | 180.4 | 75.3 | 37.1 | 112.8 | 498.9 |
| | | 1.75 | | 216.7 | 203.8 | 67.6 | 31.2 | 114.2 | 497.1 |
| | | 0.84 | | 208.9 | 201.5 | 58.7 | 28.1 | 118.2 | 426.4 |
| | | 0.35 | | 152.4 | 152.8 | 24.3 | 15.9 | 105.5 | 215.1 |
| Drew | 13.60 | 7.28 | 3 | 290.2 | 277.0 | 66.1 | 22.8 | 166.6 | 446.1 |
| | | 3.68 | | 287.0 | 280.9 | 58.3 | 20.3 | 161.4 | 448.0 |
| | | 1.84 | | 277.3 | 259.3 | 66.4 | 23.9 | 166.0 | 455.4 |
| | | 0.88 | | 332.5 | 339.0 | 58.8 | 17.7 | 170.1 | 470.6 |
| | | 0.37 | | 287.1 | 282.3 | 47.0 | 16.4 | 205.8 | 387.4 |
| Conway | 23.00 | 4.13 | 4 | 135.2 | 125.8 | 36.8 | 27.2 | 77.1 | 309.0 |
| | | 2.09 | | 133.1 | 123.3 | 34.7 | 26.1 | 81.1 | 265.7 |

**Table 1.** *Cont.*

| County | Size (i) | Sampling Density¹ (j) | ID (k) | Field STK Map Statistics per Field in mg K $kg^{-1}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Average | Median | Standard Deviation | CV(%) | Min. | Max. |
| Conway | 23.00 | 1.00 | 4 | 133.7 | 125.8 | 36.1 | 27.0 | 73.2 | 269.7 |
| | | 0.48 | | 149.8 | 139.5 | 39.7 | 26.5 | 94.7 | 339.2 |
| | | 0.22 | | 130.1 | 121.1 | 24.4 | 18.7 | 98.4 | 202.4 |
| Drew | 23.2 | 4.31 | 5 | 130.4 | 120.0 | 34.9 | 26.8 | 78.9 | 284.7 |
| | | 2.16 | | 135.9 | 124.6 | 34.5 | 25.4 | 88.2 | 289.4 |
| | | 1.08 | | 132.6 | 125.9 | 30.8 | 23.2 | 77.5 | 290.8 |
| | | 0.52 | | 140.2 | 132.8 | 33.6 | 24.0 | 89.6 | 301.5 |
| | | 0.22 | | 142.3 | 123.3 | 34.3 | 24.1 | 110.2 | 237.6 |
| Conway | 22.04 | 4.58 | 6 | 156.1 | 155.0 | 20.4 | 13.1 | 111.6 | 232.5 |
| | | 2.04 | | 155.6 | 153.7 | 19.1 | 12.2 | 116.8 | 245.8 |
| | | 1.00 | | 151.9 | 152.1 | 13.9 | 9.2 | 107.0 | 195.5 |
| | | 0.50 | | 145.4 | 145.4 | 15.0 | 10.3 | 113.8 | 189.8 |
| | | 0.23 | | 166.9 | 176.6 | 20.6 | 12.4 | 126.4 | 195.9 |
| St. Francis | 12.48 | 8.01 | 7 | 125.4 | 121.2 | 19.9 | 15.8 | 47.4 | 206.2 |
| | | 4.01 | | 124.1 | 120.5 | 17.8 | 14.3 | 85.2 | 186.8 |
| | | 2.00 | | 127.4 | 124.4 | 17.2 | 13.5 | 85.9 | 184.7 |
| | | 0.96 | | 124.2 | 121.9 | 16.3 | 13.1 | 82.9 | 170.8 |
| | | 0.40 | | 113.9 | 112.1 | 10.4 | 9.1 | 90.5 | 136.7 |
| Conway | 23.04 | 4.34 | 8 | 130.4 | 129.3 | 15.6 | 11.9 | 94.3 | 182.7 |
| | | 2.17 | | 133.2 | 131.1 | 16.6 | 12.4 | 93.3 | 195.5 |
| | | 1.09 | | 137.9 | 131.9 | 21.3 | 15.4 | 88.6 | 201.0 |
| | | 0.52 | | 139.0 | 138.2 | 15.4 | 11.1 | 94.6 | 197.9 |
| | | 0.22 | | 142.6 | 140.1 | 20.2 | 14.2 | 86.2 | 193.6 |
| St. Francis | 12.48 | 8.01 | 9 | 73.0 | 69.7 | 16.9 | 23.2 | 44.4 | 141.6 |
| | | 4.01 | | 75.5 | 69.7 | 16.0 | 21.1 | 56.9 | 145.5 |
| | | 2.00 | | 72.7 | 69.0 | 13.8 | 18.9 | 52.8 | 145.2 |
| | | 0.96 | | 75.6 | 71.0 | 15.6 | 20.7 | 56.9 | 148.5 |
| | | 0.40 | | 69.9 | 67.8 | 10.2 | 14.5 | 54.2 | 98.3 |
| St. Francis | 12.28 | 5.05 | 10 | 66.1 | 61.5 | 11.5 | 17.4 | 45.4 | 113.4 |
| | | 2.52 | | 65.1 | 62.0 | 8.3 | 12.8 | 52.8 | 96.6 |
| | | 1.22 | | 61.3 | 58.3 | 9.6 | 15.7 | 45.0 | 96.5 |
| | | 0.65 | | 67.0 | 65.7 | 6.2 | 9.3 | 52.8 | 81.6 |
| | | 0.41 | | 65.4 | 64.0 | 5.2 | 7.9 | 58.1 | 80.5 |
| St. Francis | 7.36 | 13.59 | 11 | 60.6 | 59.3 | 8.0 | 13.2 | 44.0 | 80.6 |
| | | 6.79 | | 59.8 | 59.7 | 6.6 | 11.0 | 45.9 | 78.5 |
| | | 3.40 | | 61.0 | 61.1 | 6.0 | 9.9 | 47.6 | 81.1 |
| | | 1.63 | | 57.3 | 57.1 | 6.5 | 11.3 | 42.5 | 77.7 |
| | | 0.68 | | 63.3 | 61.5 | 5.5 | 8.7 | 55.1 | 72.8 |
| Average | 18.6 | 6.23 | 1–11 | 142.7 | 135.1 | 33.8 | 22.0 | 82.6 | 253.7 |
| | | 3.10 | | 143.4 | 136.9 | 32.3 | 20.6 | 89.0 | 256.1 |
| | | 1.54 | | 144.0 | 136.7 | 31.7 | 19.9 | 85.2 | 250.0 |
| | | 0.75 | | 151.6 | 147.6 | 30.2 | 18.3 | 90.4 | 252.6 |
| | | 0.33 | | 141.1 | 135.4 | 23.5 | 15.2 | 102.6 | 193.8 |

**Note:** Field size (i) is reported in ha. See **Figure 2** for a visual example of changes in sampling density (*j*) in number of samples per hectare. Interpolating STK soil sample information to 400 $m^2$ grids using different sampling densities and *k* fields of varying sizes resulted in changes in the average, median, standard deviation, coefficient of variation, minimum, and maximum value for each field.

**Table 2.** Precentage Breakdown of state average and county-specific Mehlich-3 extractable soil-test K (STK) categories to 10 cm soil depth from the Marianna soil testing lab, 2022.

| Location | # of Samples Analyzed in 2022 | Mehlich-3 Soil Potassium (mg K $kg^{-1}$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | $<$ 61 | 61–90 | 91–130 | 131–175 | $>$ 175 | Median |
| Conway | 185 | 11 | 15 | 33 | 25 | 17 | 120 |
| Drew | 181 | 33 | 24 | 20 | 12 | 10 | 78 |
| Lincoln | 1,979 | 14 | 17 | 22 | 14 | 34 | 123 |
| St. Francis | 836 | 11 | 15 | 14 | 8 | 52 | 186 |

**Table 2.** *Cont.*

| Location | # of Samples Analyzed in 2022 | Mehlich-3 Soil Potassium (mg K kg$^{-1}$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | < 61 | 61–90 | 91–130 | 131–175 | > 175 | Median |
| State Average | 180,239 | 18 | 20 | 23 | 15 | 24 | 115 |

**Source:** Arkansas soil-test summary for samples collected in 2022 [16]. Greater detail by soil series is contained in this publication.

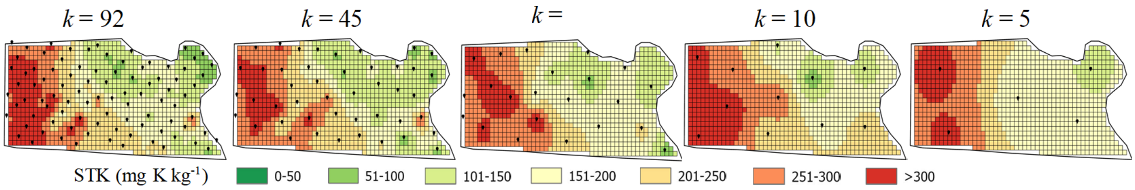



**Figure 2.** Mehlich-3 extractable soil K values in the 0–10 cm soil layer (STK) are mapped using *k* soil samples that are interpolated to 400 m$^2$ grids using ArcGIS Pro's (ESRI, Redlands, CA, USA) inverse distance weighting (radius variable 12, power 2) from highest (left) to least (right) soil sampling density. Sampling locations are shown with black dots in Field 1, Lincoln County, AR, Spring 2022.

## 2.2. Conceptual Framework of Profit-Maximizing K Fertilizer Rates

All STK maps for each field with five different sampling densities per field or 55 maps in total, were used to calculate profit-maximizing fertilizer-K rates (K$^*$s) per grid for VRT and the average profit-maximizing fertilizer-K rates (UK$^*$s) per field for URT. **Figure 3** illustrates how two different STK levels impact profit-maximizing K fertilizer rates. At low STK, a steeper yield response to K fertilizer is expected, as plants require more K than is available in the soil. Hence, the benefit of an added unit of fertilizer at low fertilizer application rates is higher than its added cost, and those benefits level off at higher fertilizer application rates as the yield response to fertilizer levels off. For both STK situations, the profit-maximizing fertilizer application rates are less than the yield-maximizing rates, as the yield response to an added unit of K fertilizer at yield maximum provides no benefit for the last added unit of K fertilizer. Applying at the yield-maximizing rate would only be profit-maximizing if K fertilizer were free. Further, while a profit-maximizing K rate of 51 kg K ha$^{-1}$ at 123 mg K kg$^{-1}$ STK is suggested in the right graph, the benefit of fertilizer application at 124 mg K kg$^{-1}$ is insufficient to cover the fertilizer application cost (**Figure 3**). Hence, if the initial STK is > 123 mg K kg$^{-1}$ in a field's grid, the K$^*$ for that grid was set to zero as the yield improvement from K$^*$ beyond that level of STK no longer sufficed to cover the fertilizer application cost and the cost of the fertilizer itself.

## 2.3. Profitability Comparisons Between VRT and URT at the Field Level

To make relative profitability comparisons between VRT and URT, we calculated partial returns, defined as the revenue from field yield less the cost of fertilizer and fertilizer application charges that varied by fertilizer application method. Other charges for growing irrigated soybean (e.g., irrigation, seed, labor, fuel, herbicides) were assumed the same regardless of fertilizer application method and thereby irrelevant for comparisons of fertilizer application method.

To obtain estimates for field yield, fundamental steps for calculating field fertilizer prescription maps, which identify the fertilizer application rate for each 400 m$^2$ grid in a field, are explained first. We calculated profit-maximizing K fertilizer application rates that STK impacted in grid ($i$), where STK varied by soil sampling density ($j$), as shown in **Figure 2** and **Table 1**, and finally field ($k$) characteristics (e.g., size, inherent average, and variance of STK). As shown in Equation (1), the profit-maximizing rate ($K^*$) with VRT application is impacted by $STK_{ijk}$ as well as the cost of fertilizer ($c_K$), the price of the crop ($P_S$), and a field's yield potential ($YP$) to meet the profit-maximizing condition where an added unit of fertilizer is justified given its yield impact and cost, using the quadratic yield response equations to K fertilizer estimated by Popp et al.[3] as in Badarch et al.[14].

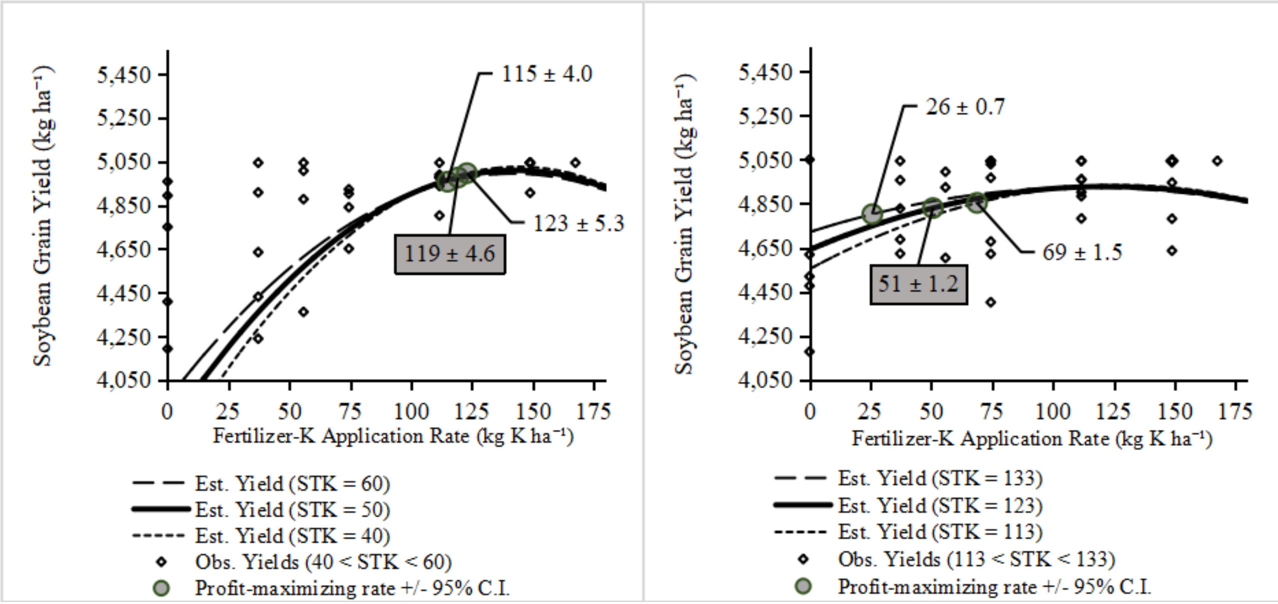



**Figure 3.** Estimated yield response and profit-maximizing K fertilizer application rates at 50 (left) and 123 ± 10 ppm (right) Mehlich-3 extractable soil K values in the 0–10 cm soil layer (STK). Observed yields are from experimental trials used to generate relative yield indices at varying STK to model yield outcomes for an example field with 5,044 kg ha$^{-1}$ irrigated soybean yield potential (Popp et al., 2020). Shaded boxes showcase profit-maximizing K rates for 50 and 123 ppm STK using 2013–2022 average soybean and muriate of potash fertilizer (0-0-60) prices of \$0.40 kg$^{-1}$ and \$1.09 kg$^{-1}$ K, respectively.

$$K_{ijk}{}^{*} = \frac{\left[\frac{c_K}{\frac{YP}{100}\cdot P_S} - \left(0.558 - 5.150\cdot 10^{-3}\cdot STK_{ijk} + 1.114\cdot 10^{-5}\cdot STK_{ijk}^2\right)\right]}{\left[2\cdot\left(-1.896\cdot 10^{-3} + 1.673\cdot 10^{-5}\cdot STK_{ijk} - 3.614\cdot 10^{-8}\cdot STK_{ijk}^2\right)\right]} \tag{1}$$

where $P_S$ = \$10.82/bu (\$0.40 kg$^{-1}$) is the average 10-year price of soybean from 2013–2022[18] to avoid unduly impacting profit-maximizing fertilizer rate by an unusually high- or low-price year. Similarly, the fertilizer cost, $c_K$ = 494.16/ton (\$1.09 kg$^{-1}$ K), was transformed from muriate of potash fertilizer (500 g K kg$^{-1}$) prices from historical Mississippi State University cost of production budgets to \$ kg$^{-1}$ K for the same period[19]. Finally, the irrigated soybean yield potential (*YP*) was set at 5,044 kg ha$^{-1}$ to reflect yields producers expect in fields not deficient in other macronutrients under good weather conditions. Note that Popp et al.[3] estimate yield response to K fertilizer using a relative yield index. Multiplying the relative yield index by *YP* yields field-specific yield responses.

To estimate the profit-maximizing URT fertilizer rate, Equation (1) is modified to utilize the average of $STK_i$ in a field *k* at varying soil sampling densities *j* as follows:

$$UK_{jk}{}^{*} = \frac{\left[\frac{c_K}{\frac{YP}{100}\cdot P_S}\left(0.558 - 5.150\cdot 10^{-3}\cdot \overline{STK}_{jk} + 1.114\cdot 10^{-5}\cdot \overline{STK}_{jk}^2\right)\right]}{\left[2\cdot\left(-1.896\cdot 10^{-3} + 1.673\cdot 10^{-5}\cdot \overline{STK}_{jk} - 3.614\cdot 10^{-8}\cdot \overline{STK}_{jk}^2\right)\right]} \tag{2}$$

The main difference between Equations (1) and (2) is that grid-level $K^*$ varies within a field with VRT, whereas $UK^*$ is applied uniformly or at the same level in each grid within a field. Nonetheless, $UK^*$ is still modeled to vary by field with changes in sampling density (**Figure 2** and **Table 1**) and by field. Hence fertilizer prescription maps showcase $K_{ijk}{}^*$ by grid within a field as $STK_{ijk}$ varies by soil map created at different sampling densities (**Figure 4**) whereas $UK^*$ is a fertilizer rate recommendation that is applied uniformly across the field.

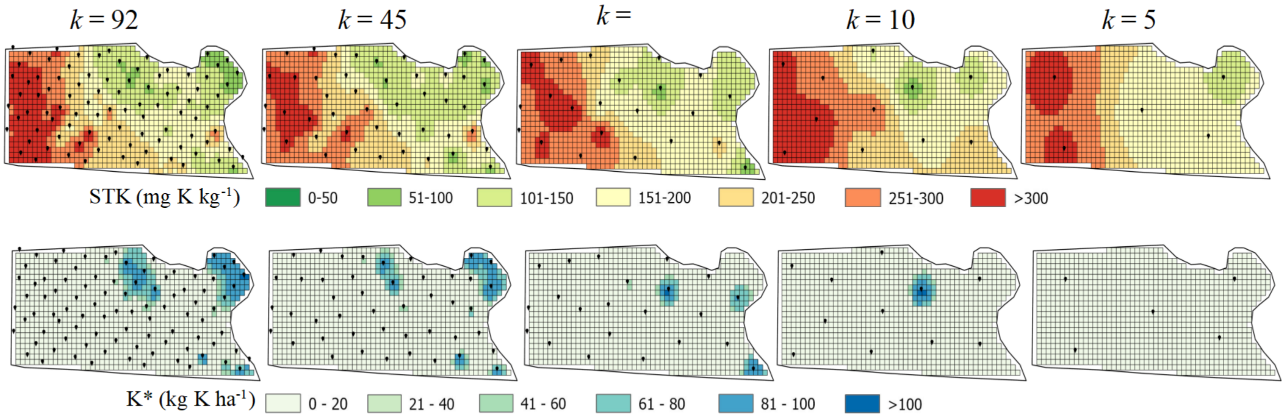



**Figure 4.** Mehlich-3 extractable soil K values in the 0–10 cm soil layer (STK) are mapped using *k* soil samples that are interpolated to 400 $m^2$ grids using ArcGIS Pro's (ESRI, Redlands, CA, USA) inverse distance weighting (radius variable 12, power 2) from highest (left) to least (right) soil sampling density in the top row. Sampling locations are shown with black dots. Corresponding profit-maximizing K fertilizer rates are mapped in the bottom row in Field 1, Lincoln County, AR, Spring 2022.

Successively removing soil sample information to measure the impacts of soil sampling density saves on soil sampling cost, but also reduces the accuracy of field STK maps and thereby impacts field yield performance, as $K^*$ based on less informed maps will be applied at higher or lower rates than the $K^*$ assessed with the highest level of STK information or the most soil samples. That is, less information leads to poorer nutrient matching between K sources, *STK*, and $K^*$ (if any), and the plants' profit-maximizing K needs.

Grid-level yield estimates ($\widehat{Y}_{ijk}$) across all sampling densities modeled for a field are based on the most accurate STK map and vary across application technology. Using Popp et al.'s [3] relative yield response coefficient estimates, field yields were calculated as the sum of all yields observed per grid as follows:

$$\begin{gathered}\widehat{Y}_{ijk} = (60.01 + 40.35 \cdot STK_{ijk} - 7.62 \\ \times 10^{-4} \cdot STK_{ijk}^2 + 0.56 \cdot K_{ijk}^* - 1.90 \\ \times 10^{-3} \cdot {K_{ijk}^*}^2 - 5.15 \\ \times 10^{-3} \cdot STK_{ijk} \cdot K_{ijk}^* + 1.67 \\ \times 10^{-5} \cdot STK_{ijk} \cdot {K_{ijk}^*}^2 + 1.11 \\ \times 10^{-5} \cdot {STK_{ijk}}^2 \cdot K_{ijk}^* - 3.61 \\ \times 10^{-8} \cdot {STK_{ijk}}^2 \cdot {K_{ijk}^*}^2)/100 \cdot YP/25\end{gathered} \tag{3}$$

where the part of the equation within parentheses predicts the relative yield index for site-specific $STK_{ijk}$ and $K^*$, and the coefficient estimates are derived from 91 site years of fertilizer rate trials. Dividing the relative yield index by 100 and multiplying by a field's yield potential led to a per-hectare yield estimate that was divided by 25 to account for the number of 400 $m^2$ grid $ha^{-1}$. Using Equations (1) and (3), field-level partial returns from VRT (*VPR*) in different fields *k* and at different sampling densities *j* were thus estimated using:

$$\begin{gathered}VPR_{jk} = \textstyle\sum_{i-1}^{n}(\widehat{Y}_{ijk,VRT} \cdot P_S - {K_{ijk}}^*/25 \cdot c_K \\ -C_{VRT}/25) - FSSC_{jk}\end{gathered} \tag{4}$$

where *n* is the number of grids (*i*) in a field (*k*), $C_{VRT}$ = \$5 $ha^{-1}$ are added VRT application charges in comparison to URT application, and $FSSC_{jk}$ are field soil sampling charges that vary by the number of samples used with different sampling densities (*j*) within each field for \$5.50 per sample as reported by Mississippi State University [19].

Field-level partial returns for URT (*UPR*), use ${UK_{jk}}^*$ from Equation (2) instead of ${K_{ijk}}^*$ in Equation (3) to arrive at yield estimates and ultimately *UPR* as follows:

$$\begin{gathered}UPR_{jk} = \textstyle\sum_{i-1}^{n}(\widehat{Y}_{ijk,URT} \cdot P_S \\ -{UK_{jk}}^*/25 \cdot c_K) - FSSC_{jk}\end{gathered} \tag{5}$$

Both $UPR_j$ and $VPR_j$ within each of the *k* fields are compared to identify the soil sampling density with the highest partial returns. This was done to guide economically optimal soil sampling densities given tradeoffs between the cost of soil sampling and the value of added information it creates. As such, identifying the highest *UPR* or *VPR* in a field across the different soil sampling densities determines what soil sampling density is economically optimal across the eleven fields evaluated.

## 2.4. Modeling Profitability Differences Between URT and VRT Using Field STK Information

The difference between *VPR* and *UPR,* converted to $ ha$^{-1}$ for profitability comparison between application technologies across fields of varying size, was calculated as follows:

$$\Delta PR_{jk} = \frac{VPR_{jk} - UPR_{jk}}{Size_k} \quad (6)$$

where a positive $\Delta PR_{jk}$ indicated VRT as the profit-maximizing choice at a particular sampling density, *j*, in a field *k*.

To determine the influence of field STK map characteristics on the profitability of VRT relative to URT, two models using multivariate regression were estimated. The first model assessed the relationship between the standard deviation of STK as impacted by the level of STK, the size of the field, and sampling density, as spatial variation in STK should impact nutrient mismatch and hence the viability of VRT in comparison to URT. The second model quantifies the impact of the average and standard deviation of STK, as well as sampling density and field size impact on profitability differences between URT and VRT, along with crop price and fertilizer cost.

$$\begin{aligned} \sigma_{STK_{jk}} &= \alpha_0 + \alpha_1 \overline{STK}_{jk} \\ &+ \alpha_2 SIZE + \alpha_3 SD_{ij} + \delta_{ij} \end{aligned} \quad (7)$$

$$\begin{aligned} \Delta PR_{ij} &= \beta_0 + \beta_1 \overline{STK}_{jk} + \beta_2 \cdot \overline{STK}_{jk}^{0.5} \\ &+ \beta_3 \sigma_{STK_{jk}} + \beta_4 {\sigma_{STK_{jk}}}^2 \\ &+ \beta_5 \overline{STK}_{jk} \cdot \sigma_{STK_{jk}} + \beta_6 SIZE \\ &+ \beta_7 SD + \beta_8 SIZE \cdot SD \\ &+ \beta_9 P_S + \beta_{10} c_K + \varepsilon_{ij} \end{aligned} \quad (8)$$

where $\overline{STK}$ and $\sigma_{STK}$ are the mean and standard deviation of STK, and *SIZE* and *SD* are the field size and sampling density, respectively, for each of the eleven fields *k* and *j* sampling densities, $P_S$ and $c_K$ are the crop price and fertilizer cost, and δ and ε are normally distributed, two-sided error terms with zero mean. Error terms were subjected to a Breusch-Pagan heteroskedasticity test to determine whether to correct standard errors of coefficient estimates using Huber-White heteroskedasticity-consistent covariances. Each model's explanatory and predictive power was judged via adjusted $R^2$, Akaike Information Criterion, F-statistic, and the number of statistically significant coefficient estimates. Further, Equation (8) used $\Delta PR_{jk}$ results that were replicated using sample years with the highest fertilizer cost and soybean price (2022), lowest fertilizer cost and near average soybean price (2016), second highest soybean price and near average fertilizer cost (2013), and low fertilizer cost and low soybean price (2018) to add predictive power to the model results and provide a sensitivity analysis on price and cost effects to add to the robustness of this modeling effort (**Table 3**).

**Table 3.** Historical soybean price and fertilizer-K cost along with estimated likelihood of lower price or cost by year.

| Year | Soybean Price ($ kg$^{-1}$) | K fertilzer ($ kg$^{-1}$ K) | Likelihood[1] |
|---|---|---|---|
| 2013 | 0.48 | 1.05 | 41% |
| 2014 | 0.39 | 1.04 | 32% |
| 2015 | 0.35 | 0.94 | 16% |
| 2016 | 0.36 | 0.75 | 3% |
| 2017 | 0.36 | 0.84 | 12% |
| 2018 | 0.32 | 0.89 | 7% |
| 2019 | 0.33 | 1.22 | 10% |
| 2020 | 0.39 | 0.98 | 27% |
| 2021 | 0.47 | 1.16 | 51% |
| 2022 | 0.53 | 2.06 | 94% |
| Avg. ('13–'22) | 0.40 | 1.09 | 36% |

**Note:** [1] Using fitted triangular distributions for soybean price (min. = 0.3, mode = 0.32, max. = 0.60 with an estimated mean of 0.41) and fertilizer cost (min. = 0.7, mode = 0.75, max. = 2.1 with an estimated mean of 1.18) based on the 10 year history of price information provided above and accounting for correlation between the two price series ($\rho$ = 0.70), we used Monte Carlo simulation in @Risk v7.6 (Pallisade Corporation, Ithaca, NY, 2016) to generate 10,000 observations of soybean prices and fertilizer cost to report the likelihood of lower soybean price and fertilizer cost than those observed in a particular year to assist with assessment of likelihood of jointly observing a lower price and lower fertilizer cost than the observed price and cost point for that year or range of years.

# 3. Results

## 3.1. Assessment of the Degree of Representation of Fields, Price, and Cost Information Analyzed

**Figure 4** provides a visual analysis of the variety of fields analyzed by plotting STK means ($\overline{STK}$) and standard deviation ($\sigma_{STK}$) of each field in addition to the summary information already provided in **Table 1**. The standard deviation of STK was directly correlated with $\overline{STK}$, field size, and sampling density (**Table 4**). Three fields each were clustered in the low and high $\overline{STK}$ and $\sigma_{STK}$ ranges. Five fields had mid-level $\overline{STK}$ (near or above the 123 mg K kg$^{-1}$ STK level required to justify a profit-maximizing fertilizer-K application, as identified in section 2.2.) with mid- or low-level $\sigma_{STK}$. In comparison to state-average STK (**Table 2**), the fields thus represent a large range of STK conditions that are deemed representative of mid-Southern agricultural fields. To what extent within-field variation in STK is representative of fields in the region is difficult to assess and considered beyond the scope of this work.

## 3.2. Profitability Implications of Low, Mid-Level, and High STK on Sampling Density and Technology Choice

Similar to Koch et al. [21], initial thoughts were that greater $\sigma_{STK}$ in a field's STK would be greater justification for VRT as more instances of grid-level mismatch between nutrient source and needs would arise with URT than VRT. We found VRT to be more profitable than URT in only two instances, fields 4 and 5, as indicated by the bold letters in the legend of **Figure 5**.

At high STK values, supplemental K fertilizer is not needed, as shown in **Table 5** with $K^*$ and $UK^*$ at or near zero application rates for most grids. Soil sources of K (STK) and supplemental K fertilizer for the high $\sigma_{STK}$ fields 1–3 are shown in **Figure 6A** to illustrate how too few grids with non-zero $K^*$ drive the economic conclusion that zero $UK^*$ with URT is more profitable than VRT. The same conclusion is observed at the least sampling density, as shown in **Figure 6B**. Lesser STK map accuracy translated to more nutrient mismatch in comparison to the most accurate and costliest STK maps—see insufficient fertilizer application in Field 1 and

excess application in Field 2—but the yield implications of this mismatch did not justify greater sampling density nor more costly VRT in comparison to URT as indicated in **Table 5**.

For fields with both $\overline{STK}$ and $\sigma_{STK}$ at mid-level (**Figure 5**), VRT proved more profitable than URT (**Figures 7A** and **7B**). Non-zero, grid-level $K^*$ covered most of the prescription maps and yield responses at the tipping point between no fertilizer application and using supplemental fertilizer were large enough to make VRT more profitable than URT (**Table 5**). Without this large VRT yield impact over URT, where zero $UK^*$ was the prescription, a situation unique to this level of STK, URT was the profit-maximizing choice in fields with the high $\overline{STK}$ (already discussed above) and the lowest $\overline{STK}$ where supplemental K needs are evident (**Figures 8A** and **8B**). Before describing low $\overline{STK}$ and $\sigma_{STK}$ results, however, referring again to **Figures 7A** and **7B** for mid-range $\overline{STK}$ and $\sigma_{STK}$, the lesser STK map accuracy played a large role in Field 5, as the high supplemental fertilizer need was left nearly undetected near the right side of the field, with fewer soil samples. This suggested a need for greater STK map accuracy for Field 5 in comparison to Field 4 and all other fields for that matter, as indicated by the bold VPR numbers in **Table 5**, which indicated the level of soil sampling accuracy needed to achieve maximum partial returns with variable rate technology. Fields 8 and 2 were the only other fields where added STK map accuracy played a role.

**Table 4.** Multivariate regression results describing the relationship between the standard deviation of soil-test K (STK)[1] and average $\overline{STK}$, field size (*SIZE*), and soil sampling density (*SD*) across eleven fields.

| Explanatory Variables[2] | Coefficient Estimate (SE)[3] |
|---|---|
| Constant | –20.15$^{***}$ (3.17) |
| $\overline{STK}$ | 0.25$^{***}$ (0.03) |
| *SIZE* | 0.62$^{**}$ (0.19) |
| *SD* | 1.19$^{*}$ (0.53) |
| F-statistic (*p*-value) | 57.22$^{***}$ (< 0.0001) |

**Table 4.** *Cont.*

| Explanatory Variables[2] | Coefficient Estimate (SE)[3] |
|---|---|
| $R^2$ | 0.77 |
| Adj. $R^2$ | 0.76 |
| AIC | 7.68 |

**Note:** [1] See Equation (7); [2] $\overline{STK}$, *SIZE*, and *SD* are average STK values, field size, and sampling density for the 55 observations available from **Table 1**; and [3] The model results revealed statistically significant heteroskedasticity in the error terms ($p$ = 0.07) using a Breusch-Pagan-Godfrey test. Standard errors were therefore corrected using Huber-White's process in EViews v9.5 [20]. ***, **, and * indicate statistical significance at $p$ < 0.001, 0.01, and 0.05, respectively. AIC is the Akaike Information Criterion, where lower values are desired to avoid over-specification.

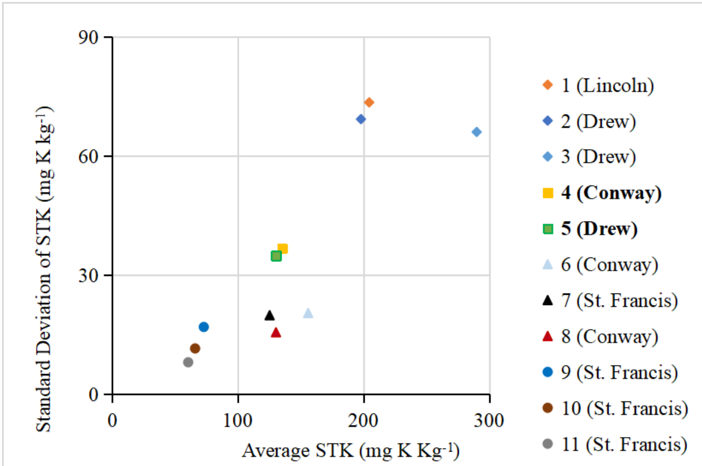



**Figure 5.** Average and standard deviation of STK from maps at the highest soil sampling density. Bold field numbers and counties represent the fields where variable rate technology was more profitable than uniform rate technology for fertilizer application.

**Table 5.** Field partial returns as a function of estimated yield response to profit-maximizing K fertilizer application per 400 $m^2$ grid using STK maps that vary by sampling density and application method, Arkansas, 2013–2022.

| Size (i) | SD (j)[c] | Avg. STK | ID (k) | Variable Rate Technology (VRT) | | | | Uniform Rate Technology (URT) | | | | EOSD[4] | Method |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | FSSC + CVRT2 | Fert. Cost | Y (kg ha-1) | VPR[3] | FSSC | Fert. Cost | Y(kg ha-1) | UPR[3] | | |
| | 2.23 | 205 | | 712 | 536 | ***4,923***[5] | 79,425 | 506 | 0 | 4,863 | 79,177 | | |
| | 1.09 | 204 | | 454 | 366 | 4,906 | 79,562 | 248 | 0 | 4,863 | 79,436 | | |
| 41.2 | 0.53 | 212 | 1 | 327 | 194 | 4,882 | 79,466 | 121 | 0 | 4,863 | 79,562 | 0.12 | URT |
| | 0.27 | 227 | | 267 | 109 | 4,872 | 79,448 | 61 | 0 | 4,863 | 79,623 | | |
| | 0.12 | 218 | | 28 | 0 | 4,863 | ***79,656*** | 28 | 0 | 4,863 | ***79,656*** | | |
| | 6.99 | 198 | | 621 | 5 | 4,877 | 27,074 | 550 | 0 | 4,876 | 27,143 | | |
| | 3.50 | 203 | | 346 | 23 | ***4,881*** | 27,351 | 275 | 0 | 4,876 | 27,418 | | |
| 14.3 | 1.75 | 217 | 2 | 209 | 8 | 4,878 | 27,486 | 138 | 0 | 4,876 | 27,556 | 0.35 | URT |
| | 0.84 | 209 | | 137 | 5 | 4,877 | ***27,557*** | 66 | 0 | 4,876 | 27,627 | | |
| | 0.35 | 152 | | 99 | 117 | 4,877 | 27,480 | 28 | 0 | 4,876 | ***27,666*** | | |
| | 7.28 | 290 | | 545 | 0 | ***4,804*** | 25,440 | 545 | 0 | 4,804 | 25,440 | | |
| | 3.68 | 287 | | 275 | 0 | 4,804 | 25,710 | 275 | 0 | 4,804 | 25,710 | | |
| 13.6 | 1.84 | 277 | 3 | 138 | 0 | 4,804 | 25,847 | 138 | 0 | 4,804 | 25,847 | 0.37 | URT |
| | 0.88 | 333 | | 66 | 0 | 4,804 | 25,919 | 66 | 0 | 4,804 | 25,919 | | |
| | 0.37 | 287 | | 28 | 0 | 4,804 | ***25,957*** | 28 | 0 | 4,804 | ***25,957*** | | |
| | 4.13 | 135 | | 638 | 904 | ***4,865*** | 42,959 | 523 | 0 | 4,686 | 42,343 | | |
| | 2.09 | 133 | | 379 | 939 | 4,862 | 43,153 | 264 | 0 | 4,686 | 42,601 | | |
| 23.0 | 1.00 | 134 | 4 | 242 | 945 | 4,858 | 43,254 | 127 | 0 | 4,686 | 42,739 | 0.22 | VRT |
| | 0.48 | 150 | | 176 | 435 | 4,778 | 43,096 | 61 | 0 | 4,686 | 42,805 | | |
| | 0.22 | 130 | | 143 | 972 | 4,859 | ***43,329*** | 28 | 0 | 4,686 | ***42,838*** | | |
| | 4.31 | 130 | | 666 | 1,023 | ***4,840*** | 42,965 | 550 | 0 | 4,654 | 42,393 | | |
| | 2.16 | 136 | | 391 | 726 | 4,783 | 43,017 | 275 | 0 | 4,654 | 42,668 | | |
| 23.2 | 1.08 | 133 | 5 | 254 | 738 | 4,787 | ***43,182*** | 138 | 0 | 4,654 | 42,805 | 1.08 | VRT |
| | 0.52 | 140 | | 182 | 475 | 4,744 | 43,116 | 66 | 0 | 4,654 | 42,877 | | |
| | 0.22 | 142 | | 144 | 752 | 4,767 | 43,084 | 28 | 0 | 4,654 | ***42,915*** | | |
| | 4.58 | 156 | | 666 | 35 | ***4,866*** | 41,955 | 556 | 0 | 4,861 | 42,049 | | |
| | 2.04 | 156 | | 358 | 13 | 4,863 | 42,252 | 248 | 0 | 4,861 | 42,357 | | |
| 22.0 | 1.00 | 152 | 6 | 231 | 36 | 4,865 | 42,379 | 121 | 0 | 4,861 | 42,484 | 0.23 | URT |
| | 0.50 | 145 | | 171 | 47 | 4,864 | 42,418 | 61 | 0 | 4,861 | 42,544 | | |
| | 0.23 | 167 | | 28 | 0 | 4,861 | ***42,577*** | 28 | 0 | 4,861 | ***42,577*** | | |
| 12.5 | 8.01 | 125 | 7 | 612 | 514 | 4,820 | 22,798 | 550 | 0 | 4,646 | 22,509 | 0.40 | URT |
| | 4.01 | 124 | | 337 | 535 | 4,800 | 22,953 | 275 | 0 | 4,646 | 22,784 | | |

**Table 5.** *Cont.*

| Size (i) | SD (j)[C] | Avg. STK | ID (k) | Variable Rate Technology (VRT) | | | | Uniform Rate Technology (URT) | | | | EOSD[4] | Method |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | FSSC + CVRT2 | Fert. Cost | Y (kg ha$^{-1}$) | VPR[3] | FSSC | Fert. Cost | Y(kg ha$^{-1}$) | UPR[3] | | |
| 12.5 | 2.00 | 127 | 7 | 200 | 431 | 4,792 | 23,154 | 138 | 0 | 4,646 | 22,921 | 0.40 | URT |
| | 0.96 | 124 | | 128 | 507 | 4,801 | 23,195 | 66 | 0 | 4,646 | 22,993 | | |
| | 0.40 | 114 | | 90 | 798 | ***4,857***[5] | ***23,219*** | 28 | 887 | 4,867 | ***23,242*** | | |
| 23.0 | 4.34 | 130 | 8 | 665 | 559 | ***4,788*** | 42,653 | 550 | 0 | 4,692 | 42,441 | 0.22 | URT |
| | 2.17 | 133 | | 390 | 493 | 4,772 | 42,842 | 275 | 0 | 4,692 | 42,716 | | |
| | 1.09 | 138 | | 253 | 402 | 4,754 | 42,904 | 138 | 0 | 4,692 | 42,853 | | |
| | 0.52 | 139 | | 181 | 198 | 4,724 | ***42,906*** | 66 | 0 | 4,692 | 42,925 | | |
| | 0.22 | 143 | | 143 | 177 | 4,713 | 42,865 | 28 | 0 | 4,692 | ***42,963*** | | |
| 12.5 | 8.01 | 73 | 9 | 612 | 105 | 4,930 | 22,425 | 550 | 110 | 4,938 | 22,459 | 0.40 | URT |
| | 4.01 | 76 | | 337 | 103 | 4,922 | 22,682 | 275 | 105 | 4,924 | 22,732 | | |
| | 2.00 | 73 | | 200 | 106 | 4,932 | 22,832 | 138 | 110 | 4,938 | 22,872 | | |
| | 0.96 | 76 | | 128 | 103 | 4,923 | 22,897 | 66 | 105 | 4,924 | 22,941 | | |
| | 0.40 | 70 | | 90 | 109 | ***4,937*** | ***22,928*** | 28 | 110 | 4,938 | ***22,982*** | | |
| 12.3 | 5.05 | 66 | 10 | 402 | 1,493 | 4,946 | 22,259 | 341 | 1,477 | 4,940 | 22,310 | 0.41 | URT |
| | 2.52 | 65 | | 232 | 1,504 | 4,947 | 22,425 | 171 | 1,477 | 4,940 | 22,480 | | |
| | 1.22 | 61 | | 144 | 1,527 | ***4,952*** | 22,511 | 83 | 1,544 | 4,954 | 22,567 | | |
| | 0.65 | 67 | | 105 | 1,493 | 4,945 | 22,550 | 44 | 1,477 | 4,940 | 22,607 | | |
| | 0.41 | 65 | | 89 | 1,509 | 4,947 | ***22,564*** | 28 | 1,477 | 4,940 | ***22,623*** | | |
| 7.4 | 13.59 | 61 | 11 | 587 | 921 | 4,956 | 13,000 | 550 | 926 | 4,957 | 13,034 | 0.68 | URT |
| | 6.79 | 60 | | 312 | 925 | 4,957 | 13,274 | 275 | 926 | 4,957 | 13,309 | | |
| | 3.40 | 61 | | 174 | 921 | 4,956 | 13,411 | 138 | 926 | 4,957 | 13,447 | | |
| | 1.63 | 57 | | 103 | 935 | ***4,961*** | 13,482 | 66 | 926 | 4,957 | 13,518 | | |
| | 0.68 | 63 | | 64 | 909 | 4,951 | ***13,520*** | 28 | 926 | 4,957 | ***13,557*** | | |

**Note:** We assume yield potential of 5,044 kg ha$^{-1}$, 10-year average soybean price (\$0.40 kg$^{-1}$), and fertilizer-K cost (\$1.09 kg$^{-1}$ K). Profit-maximizing K fertilizer rates are applied in increments of the nearest 5.6 kg K ha$^{-1}$ when modeling VRT, and uniformly, based on average field STK, when modeling URT. [1] Sampling density (*j*) is the number of soil samples per hectare (*SD*) with field size (*i*) reported in hectares (*SIZE*). See **Figure 2** for a visualization of sampling density changes in a field (*k*); [2] Sampling cost (*FSSC*) is impacted by field size and sampling density. It is the number of samples times \$5.50 per sample (\$0.50 for collection and \$5.00 for analyzing soil information). For VRT, added fertilizer application charges amount to \$5.00 ha$^{-1}$ compared to URT ($C_{VRT}$); [3] See Equation (4) for calculating field partial returns (*VPR*) using profit-maximizing, variable rate K fertilizer rates at the grid level (*K**) and Equation (5) for field partial returns (*UPR*) using the same, uniform, profit-maximizing K fertilizer rate (*UK**) for the entire field; [4] The economically optimal sampling density (*EOSD*) is the sampling density that led to maximum field partial returns for either URT or VRT as indicated in the column titled Method; and [5] Bold and italicized numbers are the maximum for a field in terms of yield (*Y*), *VPR* and *UPR* across *SD*.

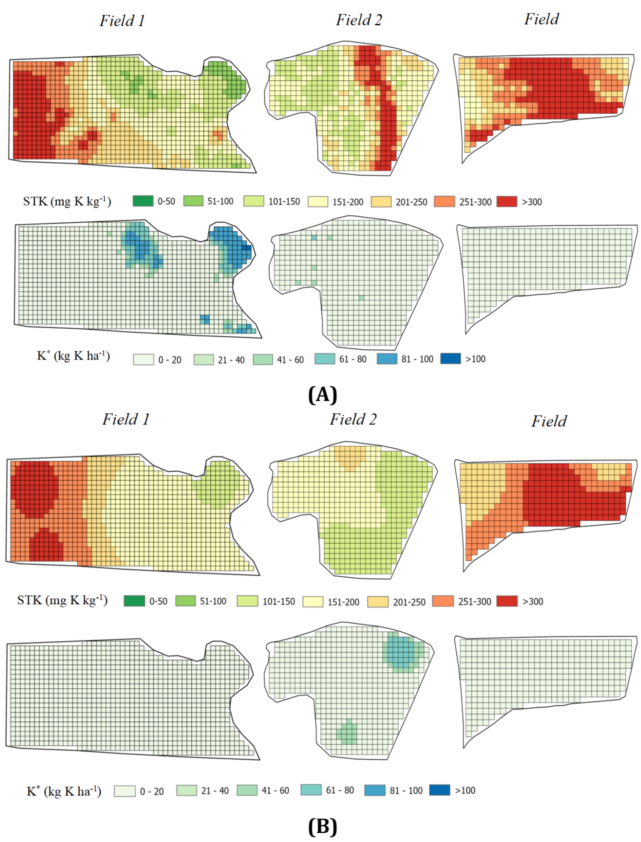



**Figure 6.** High STK value and variance fields with STK maps in the top row and corresponding K fertilizer prescription maps in the bottom row: (**A**) highest soil sampling density; and (**B**) least soil sampling density.

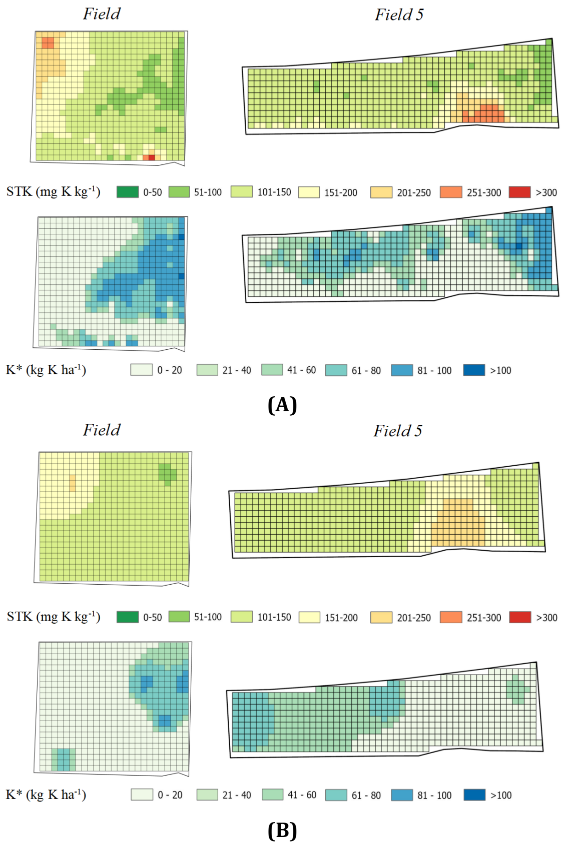



**Figure 7.** Mid-range STK value and variance fields with STK maps in the top row and corresponding K fertilizer prescription maps in the bottom row: (**A**) highest soil sampling density; and (**B**) least soil sampling density.

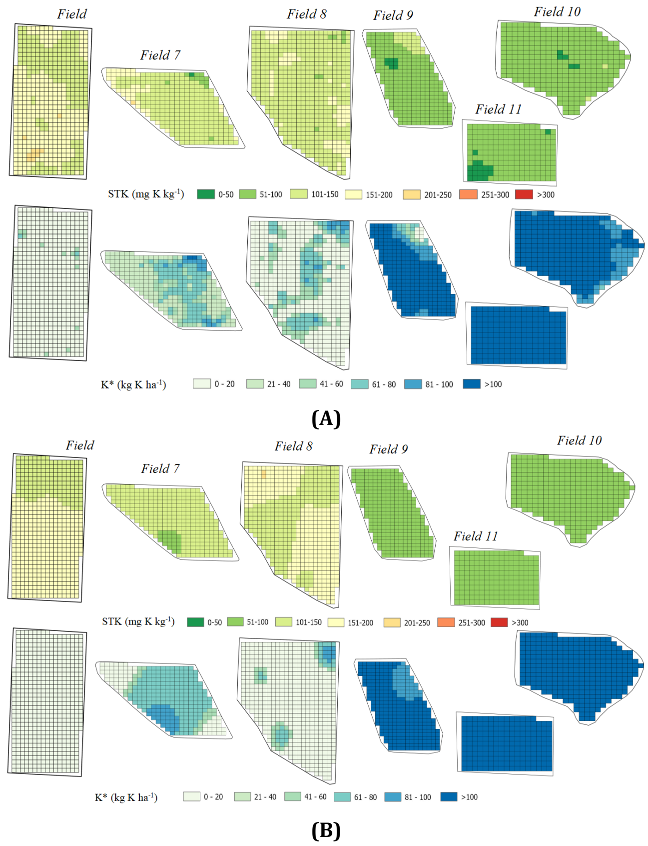



**Figure 8.** Mid-low range STK values and low-range variance fields with STK maps in the top row and corresponding K fertilizer prescription maps in the bottom row: (**A**) highest soil sampling density; and (**B**) least soil sampling density.

In fields with low $\sigma_{STK}$ (**Figures 8A** and **8B**), several interesting observations unfold. Field 6 had both low $\sigma_{STK}$ and sufficiently high $\overline{STK}$ that supplemental fertilizer use was not justified. Regardless of sampling density, this field was more profitably farmed using URT than VRT (**Table 5**).

Field 7 was more nuanced in the sense that the $\overline{STK}$ was very close to the threshold for using supplemental fertilizer, leading to a prescription map at the highest soil sampling density, which suggested moderate fertilizer use in some areas of the field, whereas $UK^*$ was uniformly zero with URT. Similar to Field 5, Field 7 demonstrated relatively large yield gains for VRT compared to URT. However, at the least soil sampling density, $\overline{STK}$ declined, making $UK^*$ with URT nonzero and economically superior to the partial returns observed with VRT. Also evident for Field 7 is nutrient mismatch with VRT when soil sampling was the least (0.4 samples ha$^{-1}$), as opposite ends of the field were flagged for higher K fertilizer rates (**Figures 8A** and **8B**). The mismatch led to greater fertilizer use with the least-accurate soil maps, where soil sampling cost savings and yield benefits outperformed greater sampling accuracy at greater cost with less yield. At the least sampling density, only five soil sampling locations were used. Had other soil sampling spots in Field 7 been picked, this field's classification from most profitably farmed with URT could have easily changed to one that could benefit from VRT with slightly more soil sampling (0.96 samples ha$^{-1}$) than the choice identified for Field 7 with bold lettering (**Table 5**), a situation that was explained in greater detail by Badarch et al.[14]. Field 8 had many grids with non-zero $K^*$, but yield gains were insufficient to outweigh the added cost of VRT vs. URT fertilizer cost and technology charges, as zero-$UK^*$ with URT was the prescription across all soil sampling densities. Finally, Fields 9–11 were most profitably farmed with URT. They were smaller fields, which partially led to lower $\sigma_{STK}$ (**Table 4**), and with low $\overline{STK}$ and $\sigma_{STK}$, estimated VRT yields were nearly the same as those of URT, given little nutrient mismatch. As such, the value proposition of VRT was not supported (not enough variation in prescription maps). Higher sampling density without much range in STK led to only greater sampling costs that were not accompanied by large yield gains over URT (**Table 5**).

### 3.3. Sensitivity Analysis of Soybean Price and Fertilizer Cost

A summary of **Table 5** findings is provided in **Table 6** to quickly ascertain optimal technology choice as impacted by the EOSD, profit-maximizing fertilizer rate, technology, and soil sampling charges, as well as yield implications. The information in the table is repeated for different soybean prices and fertilizer cost assumptions. Higher soybean prices led to greater justification of VRT. At the same time, higher fertilizer cost, *ceteris paribus*, had the opposite effect. Higher fertilizer costs lead to reduced fertilizer use and, consequently, smaller changes in fertilizer cost savings between URT and VRT. The results are robust in the sense that alternative price levels had little implications for the profitability of VRT vs. URT. Using average price and cost information, VRT was justified near the economic threshold of STK, where fertilizer use ceased to be profitable. At relatively high or low soybean prices and costs, the same fields were identified to be feasibly farmed using VRT.

### 3.4. Generalizations From Study Findings About Field Size, Soybean Price and Fertilizer Cost, Sampling Density, and STK

While the analysis of individual fields is interesting, **Table 7** suggests that some generalizations based on field size, sampling density, soybean price, and fertilizer cost are possible. Both $\overline{STK}$ and $\sigma_{STK}$ had non-linear impacts on VRT profitability that peaked near the economic $\overline{STK}$ threshold of K fertilizer use, as shown in **Figure 9** for an average-sized field using 0.5 samples $ha^{-1}$ sampling density at average soybean price and fertilizer cost. The area highlighted in green points to field characteristics where VRT profitability exceeds URT profitability by \$10 $ha^{-1}$, a threshold trigger value considered adequate for producers to pay attention to an alternative production method. The area highlighted in green mirrors the results shown in **Table 6**. Yellow and red-tinted areas, where VRT is sufficiently less profitable than URT, as indicated in the legend of **Figure 9**, pinpoint field characteristics where non-zero $K^*$ occur too infrequently to justify K fertilizer application charges, as $\overline{STK}$ is sufficiently high to obviate the need for K fertilizer, or $\sigma_{STK}$ is too low to justify the value proposition of VRT when $\overline{STK}$ and $\sigma_{STK}$ are low. Areas in **Figure 9** without a color scheme indicate $\overline{STK}$ and $\sigma_{STK}$ combinations based on **Table 4** results that suggested a direct correlation between $\overline{STK}$ and $\sigma_{STK}$. The attached spreadsheet allows the user to modify sampling density, field size, soybean price, and fertilizer cost to visualize the feasibility of VRT vs. URT fertilizer application, given the field $\overline{STK}$ and $\sigma_{STK}$ for varying scenarios they may be interested in.

**Table 6.** Summary statistics identifying the economically optimal sampling density (EOSD), profit-maximizing fertilizer method (uniform rate technology (URT) vs. variable rate technology (VRT)) and their partial return and yield differences as a function of STK and K-fertilizer under varying conditions in AR.

| Size (i) | $SD^1$ Range (j) | ID (k) | $EOSD^2$ | | $T^3$ | $\Delta PR^4$ @ $EOSD_T$ | $(\overline{STK})$ $(\sigma_{STK})^5$ @ $EOSD_T$ | FSSC + $C_{VRT}{}^6$ @ $EOSD_T$ | $UK^{=,7}$ @ $EOSD_T$ | $K^{=,8}$ (Range, Avg.) @ $EOSD_T$ | $\Delta Y^9$ @ $EOSD_T$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | URT | VRT | | | | | | | |
| 2013–2022 avg. soybean and fertilizer at \$0.40 $kg^{-1}$ and \$1.09 $kg^{-1}$ K, respectively. K threshold = 123 mg K $kg^{-1}$ | | | | | | | | | | | |
| 41.2 | 0.12–2.23 | 1 | 0.12 | 0.12 | URT | \$0.00 | 218 (56) | \$27.50 | 0 | 0–0, 0.0 | 0 |
| 14.3 | 0.35–6.99 | 2 | 0.35 | 0.84 | URT | \$7.62 | 152 (24) | \$27.50 | 0 | 0–60, 0.3 | −1 |
| 13.6 | 0.37–7.28 | 3 | 0.37 | 0.37 | URT | \$0.00 | 287 (47) | \$27.50 | 0 | 0–0, 0.0 | 0 |
| 23.0 | 0.22–4.13 | 4 | 0.22 | 0.22 | VRT | \$21.35 | 130 (24) | \$142.50 | 0 | 0–90, 38.6 | −172 |
| 23.2 | 0.22–4.31 | 5 | 0.22 | 1.08 | VRT | \$11.49 | 133 (31) | \$253.50 | 0 | 0–105, 29.1 | −133 |
| 22.0 | 0.23–4.58 | 6 | 0.23 | 0.23 | URT | \$0.00 | 167 (21) | \$27.50 | 0 | 0–0, 0.0 | 0 |
| 12.5 | 0.40–8.01 | 7 | 0.40 | 0.40 | URT | \$1.83 | 114 (10) | \$27.50 | 65 | 0–95, 58.5 | 10 |
| 23.0 | 0.22–4.34 | 8 | 0.22 | 0.52 | URT | \$2.49 | 143 (20) | \$27.50 | 0 | 0–90, 7.8 | −32 |
| 12.5 | 0.40–8.01 | 9 | 0.40 | 0.40 | URT | \$4.32 | 70 (10) | \$27.50 | 110 | 90–120, 108.9 | 1 |
| 12.3 | 0.41–5.05 | 10 | 0.41 | 0.41 | URT | \$4.79 | 65 (5) | \$27.50 | 110 | 105–115, 112.4 | −7 |
| 7.4 | 0.68–13.59 | 11 | 0.68 | 0.68 | URT | \$4.99 | 63 (5) | \$27.50 | 115 | 110–115, 112.9 | 6 |

**Table 6.** *Cont.*

| Size (i) | SD[1] Range (j) | ID (k) | EOSD[2] | | T[3] | ΔPR[4] @ $EOSD_T$ | ($\overline{STK}$) ($\sigma_{STK}$)[5] @ $EOSD_T$ | FSSC + $C_{VRT}$[6] @ $EOSD_T$ | UK*,[7] @ $EOSD_T$ | K*,[8] (Range, Avg.) @ $EOSD_T$ | ΔY[9] @ $EOSD_T$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | URT | VRT | | | | | | | |
| 2013 soybean and fertilizer at $0.48 kg$^{-1}$ and $1.05 kg$^{-1}$ K, respectively. K threshold = 131 mg K kg$^{-1}$ | | | | | | | | | | | |
| 41.2 | 0.12–2.23 | 1 | 0.12 | 1.09 | VRT | $1.90 | 204 (68) | $453.50 | 0 | 0–110, 9.3 | −46 |
| 14.3 | 0.35–6.99 | 2 | 0.35 | 0.84 | URT | $7.53 | 152 (24) | $27.50 | 0 | 0–75, 0.4 | −1 |
| 13.6 | 0.37–7.28 | 3 | 0.37 | 0.37 | URT | $0.00 | 287 (47) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 23.0 | 0.22–4.13 | 4 | 0.22 | 0.22 | VRT | $38.73 | 130 (24) | $142.50 | 0 | 0–95, 46.3 | −192 |
| 23.2 | 0.22–4.31 | 5 | 0.22 | 1.08 | VRT | $24.69 | 133 (31) | $253.50 | 0 | 0–110, 34 | −146 |
| 22.0 | 0.23–4.58 | 6 | 0.23 | 0.23 | URT | $0.00 | 167 (21) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 12.5 | 0.40–8.01 | 7 | 0.40 | 0.40 | URT | $2.08 | 114 (10) | $27.50 | 80 | 0–105, 67.7 | 21 |
| 23.0 | 0.22–4.34 | 8 | 0.22 | 1.09 | VRT | $3.30 | 138 (21) | $252.70 | 0 | 0–105, 19.1 | −69 |
| 12.5 | 0.40–8.01 | 9 | 0.40 | 0.40 | URT | $4.46 | 70 (10) | $27.50 | 115 | 95–125, 115.2 | −2 |
| 12.3 | 0.41–5.05 | 10 | 0.41 | 0.41 | URT | $4.80 | 65 (5) | $27.50 | 120 | 110–120, 118.2 | 4 |
| 7.4 | 0.68–13.59 | 11 | 0.68 | 0.68 | URT | $5.08 | 63 (5) | $27.50 | 120 | 115–120, 118.1 | 4 |
| 2016 soybean and fertilizer at $0.36 kg$^{-1}$ and $0.75 kg$^{-1}$ K, respectively. K threshold = 129 mg K kg$^{-1}$ | | | | | | | | | | | |
| 41.2 | 0.12–2.23 | 1 | 0.12 | 0.12 | URT | $0.00 | 218 (56) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 14.3 | 0.35–6.99 | 2 | 0.35 | 0.84 | URT | $7.56 | 152 (24) | $27.50 | 0 | 0–75, 0.4 | −1 |
| 13.6 | 0.37–7.28 | 3 | 0.37 | 0.37 | URT | $0.00 | 287 (47) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 23.0 | 0.22–4.13 | 4 | 0.22 | 0.22 | VRT | $29.54 | 130 (24) | $142.50 | 0 | 0–100, 47.7 | −195 |
| 23.2 | 0.22–4.31 | 5 | 0.22 | 1.08 | VRT | $17.39 | 133 (31) | $253.50 | 0 | 0–115, 34.9 | −148 |
| 22.0 | 0.23–4.58 | 6 | 0.23 | 0.23 | URT | $0.00 | 167 (21) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 12.5 | 0.40–8.01 | 7 | 0.40 | 0.40 | URT | $3.36 | 114 (10) | $27.50 | 80 | 0–105, 69.2 | 18 |
| 23.0 | 0.22–4.34 | 8 | 0.22 | 1.09 | VRT | $0.72 | 138 (21) | $252.70 | 0 | 0–105, 19.6 | −70 |
| 12.5 | 0.40–8.01 | 9 | 0.40 | 0.40 | URT | $4.60 | 70 (10) | $27.50 | 115 | 100–125, 117.1 | −6 |
| 12.3 | 0.41–5.05 | 10 | 0.41 | 0.41 | URT | $4.85 | 65 (5) | $27.50 | 120 | 110–120, 119.1 | 1 |
| 7.4 | 0.68–13.59 | 11 | 0.68 | 0.68 | URT | $5.10 | 63 (5) | $27.50 | 120 | 115–125, 119.7 | 1 |
| 2018 soybean and fertilizer at $0.32 kg$^{-1}$ and $0.89 kg$^{-1}$ K, respectively. K threshold = 121 mg K kg$^{-1}$ | | | | | | | | | | | |
| 41.2 | 0.12–2.23 | 1 | 0.12 | 0.12 | URT | $0.00 | 218 (56) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 14.3 | 0.35–6.99 | 2 | 0.35 | 0.84 | URT | $7.64 | 152 (24) | $27.50 | 0 | 0–60, 0.3 | −1 |
| 13.6 | 0.37–7.28 | 3 | 0.37 | 0.37 | URT | $0.00 | 287 (47) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 23.0 | 0.22–4.13 | 4 | 0.22 | 0.22 | VRT | $16.61 | 130 (24) | $142.50 | 0 | 0–90, 38.9 | −173 |
| 23.2 | 0.22–4.31 | 5 | 0.22 | 1.08 | VRT | $7.69 | 133 (31) | $253.50 | 0 | 0–105, 29.2 | −134 |
| 22.0 | 0.23–4.58 | 6 | 0.23 | 0.23 | URT | $0.00 | 167 (21) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 12.5 | 0.40–8.01 | 7 | 0.40 | 0.40 | URT | $1.40 | 114 (10) | $27.50 | 70 | 0–95, 58.6 | 20 |
| 23.0 | 0.22–4.34 | 8 | 0.22 | 0.52 | URT | $3.22 | 143 (20) | $27.50 | 0 | 0–90, 7.9 | −32 |
| 12.5 | 0.40–8.01 | 9 | 0.40 | 0.40 | URT | $4.48 | 70 (10) | $27.50 | 110 | 90–120, 109.2 | 1 |
| 12.3 | 0.41–5.05 | 10 | 0.41 | 0.41 | URT | $4.82 | 65 (5) | $27.50 | 110 | 105–115, 112.5 | −7 |
| 7.4 | 0.68–13.59 | 11 | 0.68 | 0.68 | URT | $5.01 | 63 (5) | $27.50 | 115 | 110–115, 113.0 | 5 |
| 2022 soybean and fertilizer at $0.53 kg$^{-1}$ and $2.06 kg$^{-1}$ K, respectively. K threshold = 115 mg K kg$^{-1}$ | | | | | | | | | | | |
| 41.2 | 0.12–2.23 | 1 | 0.12 | 0.12 | URT | $0.00 | 218 (56) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 14.3 | 0.35–6.99 | 2 | 0.35 | 0.84 | URT | $7.70 | 152 (24) | $27.50 | 0 | 0–35, 0.2 | −1 |
| 13.6 | 0.37–7.28 | 3 | 0.37 | 0.37 | URT | $0.00 | 287 (47) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 23.0 | 0.22–4.13 | 4 | 0.22 | 0.22 | VRT | $10.81 | 130 (24) | $142.50 | 0 | 0–70, 23.2 | −120 |
| 23.2 | 0.22–4.31 | 5 | 0.22 | 1.08 | VRT | $3.52 | 133 (31) | $253.50 | 0 | 0–90, 19.2 | −100 |
| 22.0 | 0.23–4.58 | 6 | 0.23 | 0.23 | URT | $0.00 | 167 (21) | $27.50 | 0 | 0–0, 0.0 | 0 |
| 12.5 | 0.40–8.01 | 7 | 0.40 | 0.96 | VRT | $5.04 | 124 (16) | $128.40 | 45 | 0–85, 23.6 | 59 |
| 23.0 | 0.22–4.34 | 8 | 0.22 | 0.52 | URT | $4.60 | 143 (20) | $27.50 | 0 | 0–75, 4.9 | −23 |
| 12.5 | 0.40–8.01 | 9 | 0.40 | 0.40 | URT | $3.57 | 70 (10) | $27.50 | 95 | 70–105, 96.6 | −9 |
| 12.3 | 0.41–5.05 | 10 | 0.41 | 0.41 | URT | $4.78 | 65 (5) | $27.50 | 100 | 90–105, 100.5 | −3 |
| 7.4 | 0.68–13.59 | 11 | 0.68 | 0.68 | URT | $4.65 | 63 (5) | $27.50 | 100 | 95–105, 101.6 | −7 |

**Note:** [1] Sampling density (*SD*) is the number of soil samples ha$^{-1}$ (*j*) with field size (*i*) reported in hectares. See **Figure2** for a visualization of sampling density changes on STK maps and attendant changes in profit-maximizing fertilizer prescription map in Field 1 and across all fields (*k*) in **Figures 6** to **8**; [2] The economically optimal sampling density (*EOSD*) is the *SD* with highest partial returns in a field (*PR*), calculated as field yield in kg ha$^{-1}$ (*Y*) times soybean price in $ kg$^{-1}$ less fertilizer cost, fertilizer application technology ($C_{VRT}$) and soil sampling charges (*FSSC*). See details in Equations (4) and (5); [3] The profit-maximizing fertilizer application technology (*T*) is the one with the highest *PR* for URT vs. *PR* for VRT at their respective $EOSD_T$, which is *T*-dependent; [4] The difference in *PR* ($ ha$^{-1}$) between URT and VRT at their respective $EOSD_T$. Note that the sign is always positive and indicates the extra profit generated by using the optimal technology noted in the prior column; [5] The average soil-test K (*STK*) and its standard deviation at $EOSD_T$; [6] Soil sampling cost is impacted by field size (*SIZE*), *EOSD* and *T*. See Equations (4) and (5); [7] The profit-maximizing K fertilizer rate, $UK^*$ in kg K ha$^{-1}$ with URT; [8] The profit-maximizing K fertilizer rates, $K^*$ in kg K ha$^{-1}$ with VRT. See **Figures 6** to **8** for spatial detail; and [9] Negative yield differential between using the *EOSD* for URT vs. using the *EOSD* for VRT suggests yield improvement with lesser nutrient mismatch and greater average application rate. A positive number is a function of lesser fertilizer use with VRT and potential nutrient mismatch due to low *SD* and low STK map accuracy.

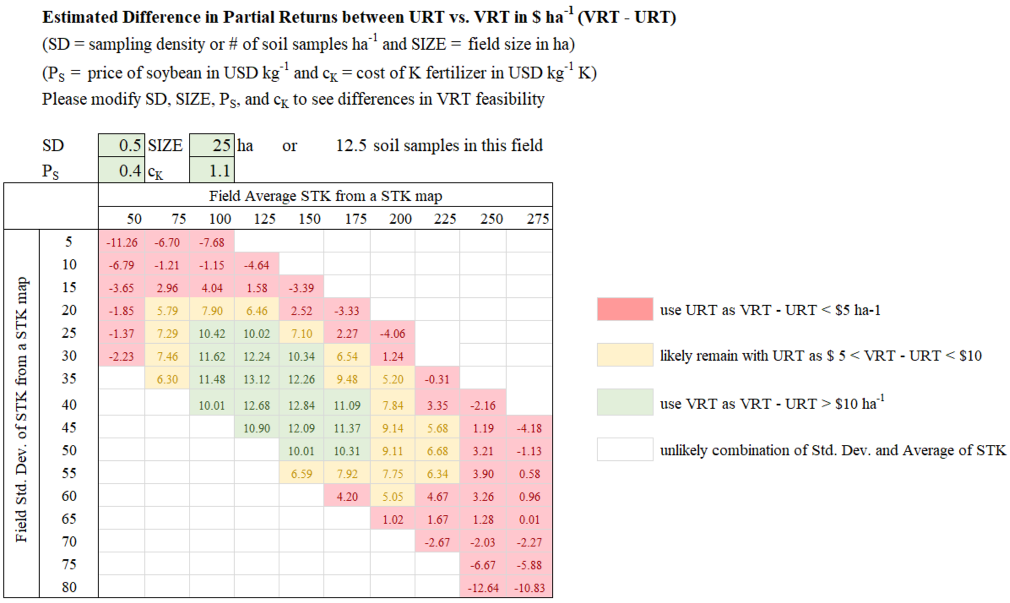
**Table 7.** Multi-variate regression results explaining profitability[1] differences between variable rate (VRT) and uniform rate (URT) fertilizer application as a function of average and standard deviation, sampling density, field size and interactions using information from eleven fields.

| Explanatory Variables[2] | Coefficient Estimate (SE)[3] |
|---|---|
| Constant | −115.12*** (22.75) |
| $\overline{STK}$ | −1.27*** (0.24) |
| $\overline{STK}^{0.5}$ | 22.19*** (4.75) |
| $\sigma_{STK}$ | 0.88*** (0.20) |
| $\sigma_{STK}{}^2$ | −0.03*** (0.002) |
| $\overline{STK}\cdot\sigma_{STK}$ | 0.01*** (0.002) |

**Table 7.** *Cont.*

| Explanatory Variables[2] | Coefficient Estimate (SE)[3] |
|---|---|
| $SIZE$ | $-0.09$ (0.07) |
| $SD$ | $-1.31^{***}$ (0.36) |
| $SIZE{\cdot}SD$ | $0.13^{***}$ (0.03) |
| $P_S$ | $35.01^{**}$ (13.44) |
| $c_K$ | $-7.48^{***}$ (2.08) |
| F-statistic ($p$-value) | $25.30^{***}$ ($<$ 0.0001) |
| $R^2$ | 0.49 |
| Adj. $R^2$ | 0.47 |
| AIC | 7.03 |

**Note:** [1] See Equation (8) for estimating the difference between VPR and UPR in USD ha$^{-1}$; [2] $\overline{STK}$ and $\sigma_{STK}$ are the sample average and standard deviation of soil-test K ($STK$), respectively. Sampling density ($SD$), is the number of soil samples per hectare, and the size of a field ($SIZE$) is in ha; [3] The model results revealed statistically significant heteroskedasticity in the error term [$p <$ 0.001 for Equation (8)] using a Breusch-Pagan-Godfrey test. Standard errors were therefore corrected using Huber-White's process in EViews v9.5 [20]. $^{***}$, $^{**}$, and $^{*}$ indicate statistical significance at $p <$ 0.001, 0.01, and 0.05, respectively. AIC is the Akaike Information Criterion, where lower values are desired to avoid over-specification. The model used 275 observations, as shown in **Tables 1** and **5**, as analyses were repeated at alternative soybean price ($P_S$) and K fertilizer cost ($c_K$) values, with partial results of those analyses in **Table 6**.



**Estimated Difference in Partial Returns between URT vs. VRT in \$ ha$^{-1}$ (VRT - URT)**
(SD = sampling density or # of soil samples ha$^{-1}$ and SIZE = field size in ha)
($P_S$ = price of soybean in USD kg$^{-1}$ and $c_K$ = cost of K fertilizer in USD kg$^{-1}$ K)
Please modify SD, SIZE, $P_S$, and $c_K$ to see differences in VRT feasibility

SD 0.5 SIZE 25 ha or 12.5 soil samples in this field
$P_S$ 0.4 $c_K$ 1.1

| Field Std. Dev. of STK from a STK map | Field Average STK from a STK map | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 |
| 5 | -11.26 | -6.70 | -7.68 | | | | | | | |
| 10 | -6.79 | -1.21 | -1.15 | -4.64 | | | | | | |
| 15 | -3.65 | 2.96 | 4.04 | 1.58 | -3.39 | | | | | |
| 20 | -1.85 | 5.79 | 7.90 | 6.46 | 2.52 | -3.33 | | | | |
| 25 | -1.37 | 7.29 | 10.42 | 10.02 | 7.10 | 2.27 | -4.06 | | | |
| 30 | -2.23 | 7.46 | 11.62 | 12.24 | 10.34 | 6.54 | 1.24 | | | |
| 35 | | 6.30 | 11.48 | 13.12 | 12.26 | 9.48 | 5.20 | -0.31 | | |
| 40 | | | 10.01 | 12.68 | 12.84 | 11.09 | 7.84 | 3.35 | -2.16 | |
| 45 | | | | 10.90 | 12.09 | 11.37 | 9.14 | 5.68 | 1.19 | -4.18 |
| 50 | | | | | 10.01 | 10.31 | 9.11 | 6.68 | 3.21 | -1.13 |
| 55 | | | | | 6.59 | 7.92 | 7.75 | 6.34 | 3.90 | 0.58 |
| 60 | | | | | | 4.20 | 5.05 | 4.67 | 3.26 | 0.96 |
| 65 | | | | | | | 1.02 | 1.67 | 1.28 | 0.01 |
| 70 | | | | | | | | -2.67 | -2.03 | -2.27 |
| 75 | | | | | | | | | -6.67 | -5.88 |
| 80 | | | | | | | | | -12.64 | -10.83 |

use URT as VRT - URT < \$5 ha-1
likely remain with URT as \$ 5 < VRT - URT < \$10
use VRT as VRT - URT > \$10 ha$^{-1}$
unlikely combination of Std. Dev. and Average of STK

**Figure 9.** Snapshot of feasibility map of attached VRT profit calculator.

## 4. Summary and Conclusion

This study analyzed actual soil test K information obtained from eleven distinct fields in Arkansas to determine an economically optimum sampling density and application technology based on estimated partial field net returns to irrigated soybean production. Modeling involved calculating profit-maximizing K fertilizer rates that could be applied at variable rates across 400 m$^2$ grids with VRT vs. a field-average profit-maximizing rate with URT. A rule of thumb about relative profitability between URT and VRT K fertilizer application emerged that centered on i) first identifying the economic threshold of soil available K or STK where supplemental K fertilizer was no longer justifiable (that threshold is lower with higher fertilizer cost and increases with higher soybean price); ii) a finding that fields with low average STK tended to also have little spatial variation in STK (thereby fields with high need for K fertilizer at low STK exhibited little variation in STK and thereby little need to change K fertilizer rate, the value proposition of VRT); iii) a similar finding that fields with high STK required little K supplementation making yield improvement, in few sub-regions of fields with VRT, insufficient to afford added charges with VRT as in Sharma and Irmak's work [22]; and iv) a realization that added STK map accuracy provided insufficient value gain to sample at densities greater than 1 sample ha$^{-1}$.

Limitations of this work are i) that a greater number of fields may lead to further insights; ii) crop differences are likely; iii) yield potential among the eleven fields was assumed the same but will vary in practice; and iv) that the predictive model for profitability differences between VRT and URT (**Table 7**) were limited to differences at a particular sampling density. While the economically optimal sampling density was always at the low end for URT, higher sampling densities were justified for VRT in several instances making the feasibility projections using **Table 7** results, as used in the attached spreadsheet, biased in favor of VRT adoption as higher soil sampling densities with URT were never justified and yet applied for comparison with VRT where greater detail led to yield benefits from less nutrient mismatch. Finally, more details about the relationship between field size, $\overline{STK}$ and $\sigma_{STK}$ would be beneficial to allow a more refined assessment of how representative this study's findings are. Also, emerging technologies using remote sensing via drones, satellites, equipment-mounted sensors, or handheld devices, in-field sensors, and in-season supplemental fertilization deserve attention as they could capture field-level nutrient variability that may be addressable at low cost without nutrient runoff.

## Author Contributions

Conceptualization, M.P., A.P., and N.S.; methodology, M.P. and S.R.; software, M.P. and B.B.; validation, M.P., A.P., and N.S.; formal analysis, M.P., B.B., and S.R.; investigation, M.P., B.B., and A.P.; resources, M.P., A.P., and N.S.; data curation, B.B., A.P., and S.R.; writing—original draft preparation, M.P. and B.B.; writing—review and editing, M.P., A.P., and N.S.; visualization, M.P. and B.B.; supervision, M.P.; project administration, M.P.; funding acquisition, M.P., A.P., and N.S. All authors have read and agreed to the published version of the manuscript.

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## Institutional Review Board Statement

Not applicable.

## Informed Consent Statement

Not applicable.

## Data Availability Statement

Data can be obtained from authors upon request.

## Conflicts of Interest

The authors declare that there is no conflict of interest.

## Abbreviations

| | |
|---|---|
| STK: | Soil-Test K in mg K kg$^{-1}$ |
| EOSD: | Economic Optimum Soil Sampling Density |
| FPR: | Field Partial Return in $ |
| VRT: | Variable Rate Technology |
| URT: | Uniform Rate Technology |

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