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ARTICLE

Remote Estimation of Sugar Beet Biomass Condition

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ABSTRACT

The presented article considers the problem of estimating the parameters of root crop biomass based on Earth remote sensing data. The underground commercial part of the biomass of this type of crops is inaccessible to optical remote sensing. The authors develop a classical approach to estimating the parameters of the state of dynamic systems based on mathematical models. In their previous works, this approach was implemented by the authors to assess crops with above-ground commercial biomass. Such crops are cereals and perennial grasses. To assess the biomass of crops with an underground commercial part, the authors proposed using three mathematical models. The first, main one, is the model of the dynamics of the biomass of a root crop, reflecting the relationship between the above-ground part of the biomass and the mass of root crops. The second is a dynamic model of the parameters of the soil environment, reflecting the removal of nutrients and moisture by the biomass of the root crop. The third is a model of optical remote sensing, reflecting the relationship between the reflectance parameters in the red and near infrared optical ranges with the parameters of the above-ground part of the biomass. Since underground biomass is inaccessible to Earth remote sensing, special requirements are imposed on the model of biomass parameter dynamics. This model must have the property of observability, which ensures the assessment of all components of the root crop biomass when probing its above-ground part. The presence of three mathematical models allows simultaneous assessment of the root crop biomass parameters and soil environment parameters with the closure of the assessment algorithm on real Earth remote sensing data. The proposed methodology and algorithms are quite applicable to other root crops, such as carrots, potatoes, etc.

Keywords: Earth Remote Sensing; Root Crops; Biomass Parameters; Mathematical Models; Estimation Algorithm

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1. Introduction

Agriculture is one of the most popular areas of using Earth remote sensing (ERS) data [1,2]. Based on this information, it becomes possible to build modern effective systems for monitoring the state of agricultural land, create automated systems for managing agricultural technologies, and solve many research problems in a new way. By means of effective monitoring systems, it is possible to solve such important national economic problems as forecasting the final result (harvest) of agricultural crops, detecting emergency situations in the fields, assessing the fertility and degree of soil degradation, accounting, inventory, and classification of agricultural land with the simultaneous formation of special large-scale plans and maps. All this makes it possible to implement effective support for management decisions in agriculture [3].

Remote sensing technologies have evolved over the years, and modern agriculture has many options to choose from in terms of both technical platforms (satellite, unmanned aerial vehicle, ground-based measuring instruments) and sensors of various physical nature (e.g., visible, multispectral, hyperspectral, thermal, radar) for collecting various agricultural data. With such technical capabilities, it is important for an agricultural producer to clearly understand the possibilities of extracting information for implementing management decisions. This is especially true for quantitative parameters of agricultural crops and the soil environment. Without such information, no management actions can be implemented. This work is aimed at solving the problem of estimating quantitative parameters of agricultural crops. The approach developed in it has already been considered in the authors' previous works [3–9]. It is based on the classical approach to estimating the parameters of the state of agricultural crops using remote sensing data, based on the use of mathematical models. When using this approach, remote sensing data are considered as an indirect measurement of the state of the object of assessment [5]. In the works of the authors, this approach was tested on grain and forage crops. The commercial part of the biomass of these crops (yield) is formed above the field surface and is accessible to remote sensing tools.

The situation changes radically when assessing the parameters of the state of root crops, the commercial part of the biomass of which is formed in the soil and is inaccessible to remote sensing tools. At the same time, the availability of such information is necessary for technology management aimed at a significant increase in the yield of these crops. The problem of assessing the parameters of the biomass of root crops can be solved in two significantly different directions. The first of them implies a significant change in the technical base of remote sensing due to the involvement of radar (RLS) sensing tools. They allow assessing the parameters of the physical state of the soil environment and, on their basis, setting and solving the problem of assessing the parameters of root crop biomass. The difficulties in implementing this direction are associated with the weak development of radar sensing tools on satellite and air platforms. The results of radar probing of soils with root crops are also insufficiently studied. The second direction is associated with the further development of the methodology of classical assessment of the state of biomass and soil environment, in which mathematical models play a leading role. This approach is considered in this paper, where the problem of assessing the parameters of sugar beet biomass is solved.

2. Development of Methods for Using Remote Sensing of the Earth in Agriculture

Methods for using remote sensing data in agriculture can be divided into two main areas. These are the assessment of non-quantitative parameters of the state of the soil and vegetation cover and the assessment of quantitative parameters of crops and the soil environment. The first of these areas has currently received much greater development than the second. Its development followed a simple and obvious path. For this, combinations of reflection parameters of preselected channels were used. Such combinations were called indices, and the so-called index space was built on them. In this space, images were built in each pixel, according to which attempts were made to isolate the object under study and assess its condition. In this case, the spectral indices used to assess the state of vegetation received the generally accepted name of vegetation indices (VI) [10–21].

In essence, the VI is a dimensionless scalar indicator reflecting the state of vegetation on a given surface area that corresponds to a pixel of the image. The number of such indices has grown rapidly and currently more than 200 variants have been obtained. No scientific and methodological basis was developed for their formation, and they were selected empirically. For this purpose, the spectrum sections most correlated with the corresponding physical parameters and properties of the studied object were selected. Most often, the two most stable sections of the plant reflectance spectra were selected for these purposes. It was found that the red zone of the spectrum (620–750 nm) accounts for the maximum absorption of solar radiation by chlorophyll, and the near infrared zone (750–1300 nm) for the maximum reflection of energy by the cellular structure of the leaf. At the same time, photosynthetic activity is manifested by greater reflection parameters in the near infrared region compared to the red region. This allows using the ratio of the reflection parameters in these areas and thus ensuring high distinguishability of the plant against the background of other elements of the soil and vegetation cover. This type of index is used to compile vegetation maps, which highlight areas not covered by vegetation.

Currently, the most widely used is the NDVI (Normalized Difference Vegetation Index), which was proposed by B.J. Rose [22]. Since its inception, it has most often been used to assess the quantitative parameters of the soil and vegetation cover. However, such attempts have most often been unsuccessful. The reasons for such failures are analyzed below. At the same time, it should be noted that this index has served as an effective indicator in detecting various types of defects and problem areas of the soil and vegetation cover. Given the lack of dimension in the NDVI index, the identified problem areas are conveniently displayed on maps in shades of different colors. At the same time, any vegetation indices do not reflect the quantitative indicators of the soil and vegetation cover, and their values largely depend on the shooting conditions and the characteristics of the equipment used. Therefore, they reflect only the relative properties of the vegetation cover, which is typical for indicators of any physical nature. Analysis of the results of using various spectral indices and the rapid growth in their number shows that to date a unified scientific and methodological approach to their formation has not yet been developed. This position does not allow to justify the emergence of new indices and analyze the efficiency of their use. The most unsuccessful by many criteria are attempts to use indices of various types to assess the quantitative parameters of the vegetation cover and biomass of agricultural crops. An analysis of a large number of works in this area showed that attempts to assess the quantitative parameters of the soil and vegetation cover based on indices are not strictly scientific, since they do not meet the requirements of modern information theory.

Thus, all indices are formed by the ratio of the sums or differences of the reflection parameters in the selected spectral ranges to their products. As a result of such operations, dimensionless scalar quantities are obtained, often normalized to unity and having positive and negative signs, depending on the ratio of the selected reflection parameters. It is impossible to estimate quantitative indicators of vegetation and soil covers by a dimensionless scalar indicator, due to the fact that restoring physical quantities from a dimensionless indicator contradicts modern information theory.

Modern information theory of assessment indicates that to assess quantitative indicators of any physical nature, a mathematical model of both the object of assessment itself and a model of the relationship of the reflection parameters from the assessed indicators is necessary [9]. The assessment procedure requires for its implementation a certain ratio between the sizes of the vector of the estimated parameters and the vector of the spectral channels of remote sensing. Any scalar indices representing combinations of spectral channels do not meet such requirements.

Analyzing the background of the use of remote sensing data in agriculture, one should pay attention to a large review article [23]. It analyzes the scientific literature for the period from 2000 to 2019. Particular attention in this analysis was paid to the use of remote sensing technologies in agriculture at all stages of production. This analysis was aimed at improving the scientific understanding of the potential of remote sensing.

Thus, if at the beginning of its development, the attention of researchers was focused on monitoring the physical parameters of soils and plant diseases, then recently the number of works on information support for management decisions at all stages of agricultural production has noticeably increased. At the same time, the authors did not touch upon the methods on which previous studies were based, and did not give comprehensive recommendations on how to use remote sensing data in the best possible way. The authors focused on the characteristics of the types and platforms of remote sensing sensors.

Conducting an analysis of the information and technical base of remote sensing, the authors paid special attention to the possibility of assessing the yield of agricultural crops using remote sensing data. At the same time, the authors' remark that in order to obtain such an assessment, it is necessary to attract additional information on influencing factors and introduce empirical or mechanistic models is very important.

The authors' conclusion is important, indicating that the use of remote sensing provides a timely and non-destructive approach to identifying, quantifying and mapping stresses in agricultural crops. Despite the very large volume of analyzed sources, the authors did not touch upon the scientific and methodological foundations of the problem of assessing the quantitative parameters of the state of agricultural crops based on remote sensing data. At the same time, it can be argued that this work is in many ways a prologue to solving this problem.

According to the authors, four methodological categories are mainly used to solve this problem today:

1. Parametric regression methods. Here, the estimated quantitative parameter is associated with spectral indices by a regression model, most often linear.

2. Nonparametric regression methods or controlled data methods. Here, the regression model directly links the specified spectral data with the estimated variable. Unlike parametric regression methods, here it is necessary to make an implicit choice regarding the spectral indices and the fitting function. Nonparametric methods are divided into linear and nonlinear regression methods.

3. Methods of inversion of physical models. These are physically based algorithms that reflect physical laws that reflect cause-and-effect relationships between physical variables.

4. Hybrid regression methods. The hybrid method combines elements of nonparametric regression and physically based models.

The boundaries of the above methods are not clearly defined, since spectral indices are used as input variables for both parametric and nonparametric methods.

Most of the above methods are not intended for estimating quantitative parameters of vegetation cover. This is due to the fact that regression models of any kind are characterized by a high degree of parameter uncertainty, which entails large errors in estimating quantitative parameters, often exceeding 50%.

Higher accuracy can be achieved using nonlinear nonparametric methods. This is achieved by using probabilistic approaches such as Gaussian process regression. Hybrid regression methods based on combining regression models with a machine learning algorithm can overcome the problem of spectral information processing speed.

3. Materials and Methods

3.1. Mathematical Models of the Object of Assessment

When choosing a mathematical model of the dynamics of root crop biomass, the following features should be taken into account.

The growth of leaves and roots are closely interrelated, but the dynamics (course) of the increase in these components of biomass during the growing season is not the same. Thus, in the second half of the growing season, the growth rate of tops can significantly lag behind the growth rate of roots. At the beginning of the growing season, there is an intensive growth of the leaf apparatus and the feeding root system. Later, in July-August, root crops grow more intensively.

In this regard, at the beginning of the growing season, the mass of leaves is several times greater, and by the end of the growing season, on the contrary, the mass of roots exceeds the mass of tops. By harvesting, leaves make up 1/3–1/2 of the root crop harvest.

It has been established that the more leaf surface per unit of root crop mass at the beginning of the growing season, the higher the mass of the root crop of such a plant by the time of harvesting, i.e., the higher the yield. Therefore, it is necessary to do everything so that the plants have optimal, but not excessively developed tops throughout the growing season. The mass of leaves usually reaches its maximum by August and then gradually decreases, while the growth of the root crop and the accumulation of sugar in it continue continuously until harvesting. **Figure 1** shows the growth process of sugar beet plants, where the development phases differ only in the number (density) of leaves, i.e. the density of the above-ground biomass.

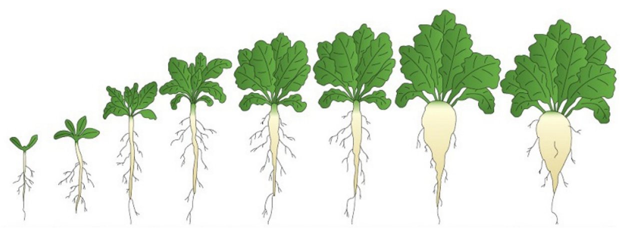

Figure 1. Phenophases of sugar beet development.

In **Figure 1**, individual phenophases of crop development differ in the number of leaves, from 2 leaves on the 12th day, to complete closure of the tops on the 40th day and full maturity of root crops on the 50th day.

The density of the above-ground biomass always corresponds to a certain biomass of root crops. The model must take into account the influence of all the main factors of growth and development, which include nutrients and external meteorological factors, as well as the features of vegetation listed above. Among all the possible parameters of the state of the biomass of root crops, we distinguish the total and raw biomass of the above-ground part and the mass of root crops. In this case, the model of the dynamics of these parameters can be presented in expanded form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}b&b&b\\b&b&b\\0&0&0\end{bmatrix}\begin{bmatrix}d_N(t)\\d_K(t)\\d_P(t)\end{bmatrix} - \begin{bmatrix}k_1&k_2\\0&0\\0&0\end{bmatrix}\begin{bmatrix}t-t_0\\(t-t_0)^2\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\d_{31}&d_{32}&d_{33}&d_{34}\end{bmatrix}\begin{bmatrix}v_N(t)\\v_K(t)\\v_P(t)\\v_4(t)\end{bmatrix} + \begin{bmatrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\0&0&c_{33}\end{bmatrix}\begin{bmatrix}f_1(t)\\f_2(t)\\f_3(t)\end{bmatrix} + \begin{bmatrix}\xi_1(t)\\\xi_2(t)\\\xi_3(t)\end{bmatrix} \quad (1)$$

or in symbolic vector-matrix form

$$\begin{gathered} \mathrm{X} = \mathrm{AX}(t) + \mathrm{Bd}(t) + \mathrm{DV}(t) - \mathrm{K}\varphi(t-t_0) + \mathrm{CF}(t) + \mathrm{Z}(t) \\ t \in (0, T), X(0) = X_0 \end{gathered} \quad (2)$$

where: t is the vegetation period, days; x_1, x_2, x_3 are the average field-area value of the total biomass of crop leaves (tops), raw above-ground biomass of crop leaves (tops), total biomass of crop root crops, cwt·ha^{-1}; d_N, d_K, d_P are the average field-area rates of foliar application of nitrogen, potassium and phosphorus fertilizers, respectively, kg·ha^{-1}; v_N, v_K, v_P is the average field-area content of the active substance, respectively, nitrogen, potassium and phosphorus in the soil, kg ha^{-1}, v_4 is the average field-area content of soil moisture, mm; $\varphi(t–t_0)$ is a vector whose components are a linear and a quadratic function of the growing season, which take into account the different rates of growth of the tops and root crops; t_0 is the moment of the growing season, from which the biomasses of the tops and root crops grow at different rates; f_1 is the average daily air temperature, °C; f_2 is the average daily solar radiation, W m^{-2}, f_3 is the average daily precipitation intensity, mm; ζ_1, ζ_2, $\zeta 3$, ζ_4 are random modeling errors that take into account unobservable and unaccounted for factors, which are random processes with zero means and variances $\sigma^2{}_1$, $\sigma^2{}_2$, $\sigma^2{}_3$, $\sigma^2{}_4$; a_{11}–a_{44} are the parameters of the dynamic matrix of model (1), (2); b_{11}–b_{23} are the parameters of the control transfer matrix of model (1), (2); k_{31}–k_{44} – parameters of the matrix of the relationship between the parameters of the state of the crop biomass and the soil parameters; c_{11}–c_{43} – parameters of the matrix of the transfer of climatic disturbances of model (1), (2).

Model (1), (2) is the main block of parameters of the state of the root crop. In addition to this block, it is necessary to introduce into consideration the model of parameters of the state of the soil environment (SE). The expanded form of this model has the following form [4,5]:

$$\begin{bmatrix} \dot{v}_N \\ \dot{v}_K \\ \dot{v}_P \\ \dot{v}_4 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} v_N(t) \\ v_K(t) \\ v_P(t) \\ v_4(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_N(t) \\ D_K(t) \\ D_P(t) \\ D_4(t) \end{bmatrix} + \\ + \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ 0 & 0 & c_{33} \\ c_{41} & c_{42} & 1 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} - \begin{bmatrix} m_{11} & 0 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & 0 & m_{33} \\ 0 & m_{42} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (3)$$

or compact symbolic form

$$\dot{V} = A_V V(t) + B_V D_V(t) + C_V F(t) - MX(t) \quad (4)$$

where D_N, D_K, D_P, D_4 are the average application rates of nitrogen, potassium, phosphorus, kg × m^2, and irrigation rates, mm, for the field area.

3.2. Estimation of Sugar Beet Biomass Parameters

Estimation of crop biomass parameters involves a comparison of estimated and actually observed parameters. When using remote sensing data, reflection parameters are observed in the used ranges of technical sensing tools. Such a comparison is only possible with the introduction of a remote sensing model. The physical basis of such models is the laws of reflection from the soil and vegetation cover, which have an exponential form [10,24]. When expanding exponential components into power series, such a model has the following form [3,5].

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} p_{01} & p_{11}x_1 & p_{12}x_2 & p_{13}x_1^2 & p_{14}x_2^2 & p_{15}x_1^3 & p_{16}x_2^3 \\ p_{02} & p_{21}x_1 & p_{22}x_2 & p_{24}x_2^2 & p_{24}x_2^2 & p_{24}x_2^2 & p_{26}x_2^3 \end{bmatrix} \quad (5)$$

$$Z(P, X) = P\,W(X) + \xi \quad (6)$$

where: $Z^T = [z_1\ z_2]$ is the vector of reflection parameters for the spatial coordinate in the visible range (400–700 nm) (z_1) and in the near infrared range (750–950 nm) (z_2);

$P = \begin{bmatrix} p_{01} & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{02} & p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \end{bmatrix}$ is the matrix of model parameters, is a $W(X) = [1 \quad x_1(y, h) \quad x_2(y, h) \quad x_1^2(y, h) \quad x_2^2(y, h) \quad x_1^3(y, h) \quad x_2^3(y, h)]$ vector function, where the arguments are the parameters of the crop state, $\xi^T = [\xi_1 \quad \xi_2]$ is the vector of errors in modeling remote sensing with zero mean and the covariance matrix K_z.

The presence of the remote sensing model (4), (5) allows us to form estimates of the reflection parameters in the spectral ranges used, compare them with real remote sensing data and, based on such a comparison, form estimates of the biomass parameters in real time. Such a procedure for restoring the full vector of biomass state parameters (ground and underground parts) based on remote sensing data observations is possible only for a system that includes models (2), (4) that have observability properties [25].

In simple terms, the observability property can be defined as the ability to determine the state vector X of model (1) from the vector Z of model (6) over a finite time interval t_0–t_1 (($t_0 > t_1$). This property is determined by the structure of models (2), (6).

This property is determined by the structure of models (2), (4). Let us introduce into consideration the transition matrix for model (2)

$$\dot{\Phi}(t) = A\,\Phi(t), t \in (0, t), \Phi(0) = E \quad (7)$$

where E is the identity matrix.

The observability condition of the system (2),(4) has the following form

$$\det\left[\int_0^\tau \Phi^T \left(\frac{\partial Z(P,X)}{\partial X}\right)^T \left(\frac{\partial Z(P,X)}{\partial X}\right) dt\right] \neq 0 \quad (8)$$

where det[...] is the determinant of the matrix.

The absence of zero terms in the matrices A and $\frac{\partial Z(P,X)}{\partial X}$ indicates that the determinant of matrix (8) is different from zero, i.e. the matrix is non-singular.

More simply, this condition can be interpreted as the absence in matrix A of columns, all elements of which are equal to zero.

As shown in [3–5], the classical theory of estimating the parameters of the state of agricultural crop biomass is based on the integration of a priori information generated by a mathematical model of biomass dynamics and a posteriori information contained in remote sensing data.

In this case, the estimation procedure for system (2), (6) has the following form [25,26].

$$\dot{\hat{\mathrm{X}}}(t) = \mathrm{A}\hat{\mathrm{X}}(t) + \mathrm{D}\hat{\mathrm{V}}(t) + \mathrm{CF}(t) + \mathrm{R}(t)\frac{\partial \mathrm{W}^{\mathrm{T}}(\mathrm{P},\hat{\mathrm{X}})}{\partial \hat{\mathrm{X}}}$$

$$\mathrm{K}_z^{-1}\left(Z(t) - \mathrm{W}^{\mathrm{T}}(\mathrm{P},\hat{\mathrm{X}})\right)\dot{\mathrm{R}}(t) = \mathrm{R}(t)\mathrm{A}^{\mathrm{T}} + \mathrm{AR}(t) - \tag{9}$$

$$\mathrm{R}(t)\frac{\partial \mathrm{W}^{\mathrm{T}}(\mathrm{P},\mathrm{X})}{\partial}\mathrm{K}_z^{-1}\frac{\partial \mathrm{W}(\mathrm{P},\hat{\mathrm{X}})}{\partial \hat{\mathrm{X}}}\mathrm{P}^{\mathrm{T}}\mathrm{R}(t)$$

matrices of estimation errors, having a dimension corresponding to the vectors of biomass parameters of model (2).

The implementation of the estimation algorithm (9) faces the problem of estimating the vector of parameters of the state of the soil environment, which is an important component of the algorithm. Due to the fact that the parameters of the state of the soil environment in turn depend on the parameters of the state of the crop due to the removal of nutrients and moisture, such a connected estimation can be implemented according to the following computational scheme:

Step 0. The initial value of the vector of parameters of the state of the soil environment is set constant for the entire considered interphase period V_0, $t \in (T_{j-1}, T_j)$, the cyclic variable $i = 0$ is set.

Step 1. The system (8) of the estimation algorithm is solved, an intermediate estimate of the vector of parameters of the state of the crop is formed $\hat{\mathrm{X}}_i(t)$.

Step 2. The current value of the efficiency criterion of the general estimation procedure is calculated

$$I_i = \int_{T_{j-1}}^{T_j} (Z(t) - \mathrm{H}(\hat{\mathrm{X}}_i(t))^{\mathrm{T}}(Z(t) - \mathrm{H}(\hat{\mathrm{X}}_i(t)),$$

where $\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ is the output matrix.

Step 3. The current estimate of the vector of parameters of the state of the crop $\hat{\mathrm{X}}_i(t)$ is substituted into model (3) and the current estimate of the vector of parameters of the state of the soil environment V*i* (t) is formed.

Step 4. The current estimate of the vector of parameters of the state of the soil environment Vi (t) is substituted into the estimation algorithm (8) and an intermediate estimate of the vector of parameters of the state of the crop is formed for the next iteration .

Step 5. The current value of the efficiency criterion of the general evaluation procedure for iteration i+1 is calculated,

$$I_{i+1} = \int_{T_{j-1}}^{T_j} (Z(t) - \mathrm{H}(\hat{\mathrm{X}}_{i+1}(t))^{\mathrm{T}} (Z(t) - \mathrm{H}(\hat{\mathrm{X}}_{i+1}(t)).$$

Step 6. The current iteration of the efficiency criterion is calculated $\delta I_{i+1} = I_{i+1} - I_i$.

If $\delta I_{i+1} \leq \Delta$, then STOP, otherwise go to step 3, until the condition is met $\delta I_{i+1} \leq \Delta$.

4. Results

4.1. Identification of Mathematical Models and Estimation of Biomass Parameters

The accuracy and reliability of assessment methods based on mathematical models largely depend on the quality of the models themselves. Therefore, the process of their identification based on real data is of particular importance. For these purposes, the fields of the experimental biopolygon of the Agrophysical Institute (St. Petersburg, Russia) were used during 2021–2024. 10–12 small plots of 10–15 m^2 each were selected on the agricultural field. From these plots, employees of the analytical laboratory of the institute simultaneously collected samples of plant biomass of the above-ground and underground parts, as well as soil samples with subsequent analysis of the content of nutrients and moisture in it. In parallel with sampling using UAVs, remote sensing of the entire field area, including control plots, was carried out. Sessions to obtain information about the real state of the object of assessment were carried out every three days. At the preliminary stage of the research, such procedures were purely identification in nature, and in real time they are designed to adapt the system to real changes in the parameters of mathematical models. Without performing such procedures, it is impossible to reliably assess the quantitative indicators of crops of any crops. Identification algorithms, due to their complexity and many possible options, deserve separate consideration and are not discussed in detail here. However, the main features of such algorithms that were used in this work should be disclosed. This will ensure the repeatability of the results by other researchers. In connection with the use of two types of mathematical models for their identification based on experimental data, two types of algorithms were used.

For the static model of remote sensing (5), (6), one of the variants of the least squares method (LSM) is used [27]

$$\begin{aligned} G(k) &= R(k-1)W(k) \\ z(k) &= W^{\mathrm{T}}(k)G(k) \\ E(k) &= Y(k+1) - \mathrm{P}^{\mathrm{T}}(k-1)W(k) \\ \mathrm{P}(k) &= \mathrm{P}(k-1) + \tfrac{1}{1+z(k)}G(k)E^{\mathrm{T}}(k) \\ R(k) &= R(k-1) - \tfrac{1}{1+z(k)}G(k)G^{\mathrm{T}}(k) \end{aligned} \tag{10}$$

in which the variables of the algorithm are related to the variables of the model (5):

- output variables, which are components of the vector Y: $y_1 = z_1, y_2 = z_2$;

- input variables, which are components of the vector W: $w_1 = 1$, $w_2 = x_1$, $w_3 = x_2$, $w = x_{21}$, $w = x_{22}$, $w = x_{31}$, $w = x_{32}$.

The estimated parameters of the model (5), which are components of the matrix P, are designated through the variables of the algorithm (10) as follows:

$p_1 = p_{01}$, $p_2 = p_{11}$, $p_3 = p_{12}$, $p_4 = p_{13}$, $p_5 = p_{14}$, $p_6 = p_{15}$, $p_7 = p_{16}$, $p_8 = p_{02}$, $p_9 = p_{21}$, $p_{10} = p_{22}$, $p_{11} = p_{23}$, $p_{12} = p_{24}$, $p_{13} = p_{25}$, $p_{14} = p_{26}$;

R, G, E, z are the intermediate variables of the algorithm; k is the iteration variable of the algorithm, which coincides with the serial number of the experimental data record used to identify the model (5).

More complex in its algorithmic approach is the identification of dynamic models of the form (2). Here, the general principles of control theory are used. They allow us to somewhat expand the identification capabilities by including in the estimation algorithm, in addition to the parameters of the dynamic model, the initial conditions for models (2), which are often a priori unknown. At the same time, the choice of initial conditions for any dynamic model can have a decisive effect on the accuracy of identification.

To proceed to the identification algorithm, it is necessary to introduce a common vector of model parameters P. For this, the parameters of models (2) which are components of the matrices, are sequentially designated row by row as components of the vector P. In addition, it is necessary to introduce a criterion for the quality of identification

$$I_0 = \frac{1}{T}\int_0^T \left(Y(t) - X(\widehat{P},t)\right)^{\mathrm{T}} \left(Y(t) - X(\widehat{P},t)\right) dt \tag{11}$$

where Y(t) is the vector of experimental values of the variable models, *X, t* is the daily time belonging to the interval (0,T).

To find the minimum of the quality criterion (11) for the vector of unknown parameters P and the initial conditions X_0, it is necessary to formulate and solve a two-point boundary value problem (TBP) [3]. To do this, it is necessary to introduce the Hamiltonian of the system:

$$\begin{aligned} H(t) = &\left(Y(t) - \widehat{X}(\widehat{P},t)\right)^{T} \left(Y(t) - \widehat{X}(P,T)\right) + \\ &\Psi^{\mathrm{T}}(t)\Phi(X,F,P,t) \end{aligned} \tag{12}$$

where Ψ is the vector of conjugate variables, $\Phi(X,F,P,t)$ is the operator of the mathematical model (2).

Based on the expressions for the Hamiltonian of the system, it is possible to obtain relationships between all states and parameters of the model (2)

$$\begin{aligned} \Psi(t) &= -\frac{\partial H}{\partial X(t)}, t \in (T,0), \Psi(T) = 0 \\ \Gamma(t) &= -\frac{\partial H}{\partial P(t)}, \Gamma(T) = 0 \end{aligned} \tag{13}$$

The sequence of operations for estimating unknown parameters and initial conditions of the model is as follows.

Step 1. When introducing a cyclic variable j, the initial values of the parameter vector P_{j0}, X_{j0} are specified.

Step 2. The model Equation (2) $X_{j(t)}$ is solved.

Step 3. According to relations (13) the variables $\Psi_j(0)$, $\Gamma_j(0)$ are determined.

Step 4. The optimal step length Δ^* is determined in the procedure $P_{j+1} = P_j - \Delta^*_j\Gamma_j(0)$ as a one-factor minimization of the quality criterion.

Step 5. With the optimal step length, a new value of the parameter vector $P_{j+1} = P_{j-}\Delta^*_j\Gamma_j(0)$ and initial conditions $X_{0,j+1} = X_{0,j} - \Delta^*_j \Psi_j(0)$ are obtained.

Step 6. Solve the equation for the model (2).

Step 7. Find new gradients $\Psi_{j}(0)$, $\Gamma_j(0)$.

Step 8. Determine the direction of the conjugate gradients

$$\widetilde{G}_{j+1} = -\Gamma(0)_{j+1} + \frac{\|\Gamma(0)_{j+1}\|^2}{\|\widetilde{G}(0)_j\|^2}\widetilde{G}_j \qquad (14)$$

at $j = 1$, $\widetilde{G}_j = \Gamma(0)_0$;

$$g_{j+1} = -\Psi(0)_{j+1} + \frac{\|\Psi(0)_{j+1}\|^2}{\|g(0)_j\|^2} g_j \qquad (15)$$

at $j = 1$, $g_j = -\Psi(0)_0$;

return to step 4.

Figure 2 shows the results of identification of the mathematical model of remote sensing, and **Figure 3** shows the process of identification of the model of biomass dynamics up to the moment of time when the rate of increase of the biomass of root crops lagged behind the rate of increase of the biomass of tops.

Figure 4 shows the process of identifying the biomass dynamics model after the moment of time when the rate of increase in root crop biomass outpaced the rate of increase in tops biomass. As can be seen from the presented graphs, the mathematical models used together with the identification algorithms have sufficient accuracy (±10%) and stability for constructing evaluation algorithms.

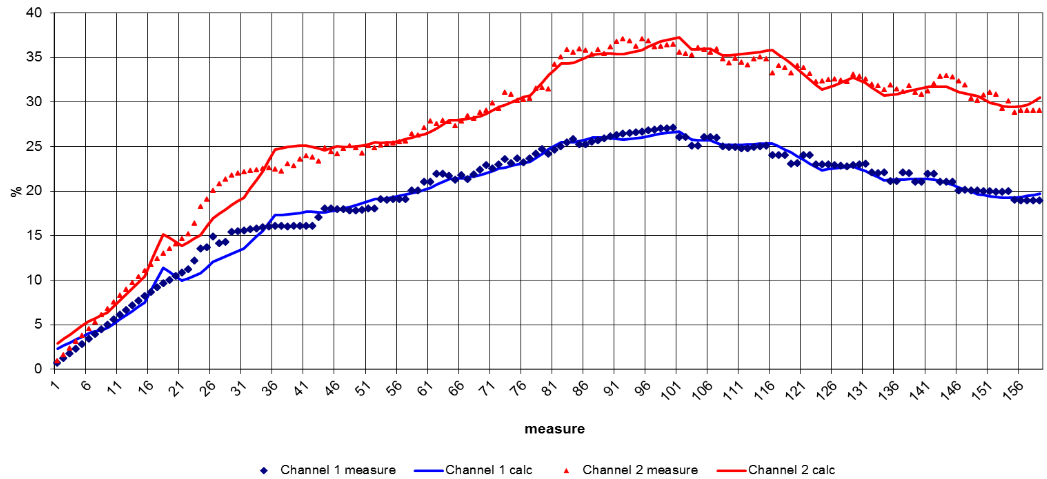

Figure 2. The process of identifying the remote sensing model.

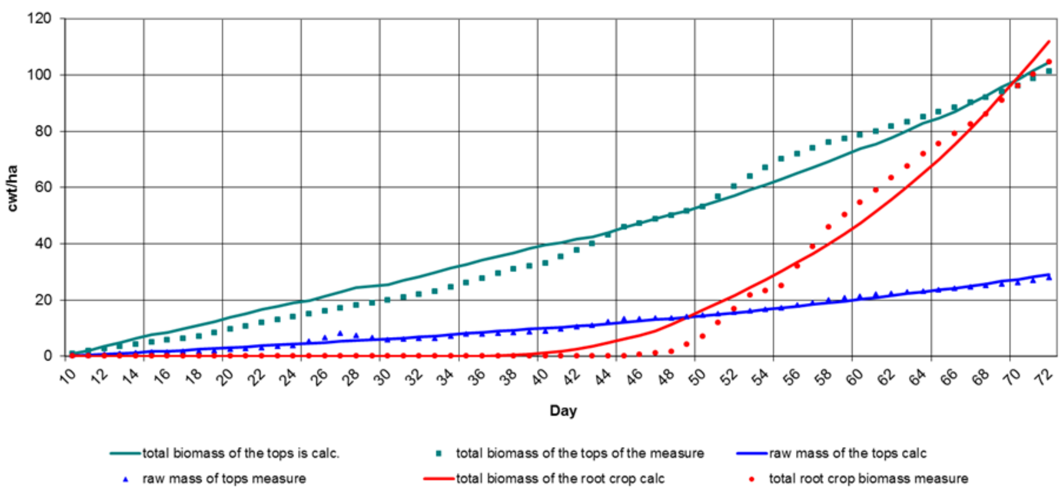

Figure 3. The process of identifying the model of beet biomass dynamics in the area of the lag in the rate of increase of root crop biomass from the biomass of tops.

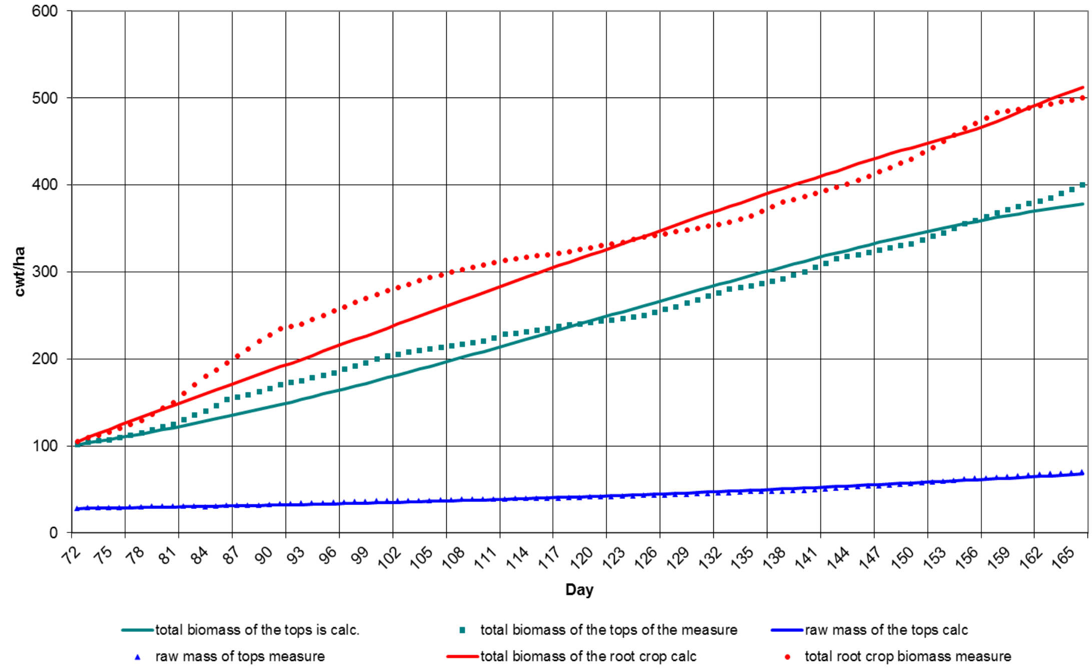

Figure 4. The process of identifying the model of beet biomass dynamics in the area of advance of the rate of increase of root crop biomass and tops biomass.

4.2. Evaluation of Beet Biomass Parameters

It was carried out according to the evaluation algorithm (9) based on real data from remote sensing of the Earth.

Figure 5 shows the process of estimating the parameters of beet biomass over the entire vegetation interval. The estimation error, including the mass of root crops, does not exceed 10%.

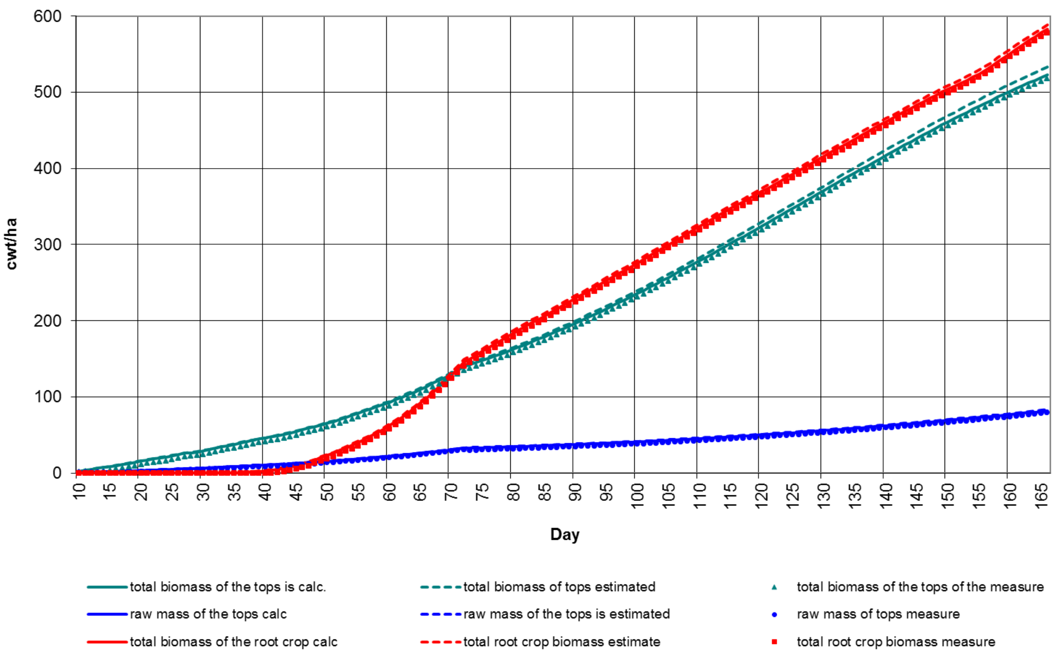

Figure 5. The process of assessing the parameters of beet biomass throughout the entire vegetation period.

5. Discussion

The development of methods for estimating the mass parameters of agricultural crops in the direction of root crops is an important turning point in the general theory of using Earth remote sensing data in agriculture. The extension of these methods to crops with a similar biomass structure, carrots, beets and radishes, will not encounter serious difficulties. Here, mathematical models with a similar structure will be used, the differences will affect only the parameters of the models themselves. The bottleneck of the estimation methods based on the use of mathematical models is the accuracy of identifying such models. The problem is that the identification of such models is carried out by periodically sampling plant biomass and soil samples from sample points of the field area. The accuracy of model identification largely depends on the number of sampling points by area and the frequency of sampling over time. Therefore, additional research is required to optimize the number of sampling points and their frequency during the growing season of the crop.

Difficulties in implementing the method will arise when estimating the biomass of potatoes. This is due to the fact that this crop has a complex morphological structure of biomass and several phenophases of crop development. A more complex dynamic model of potato crop biomass dynamics should be substantiated here, taking into account the change in morphological structure over time. Naturally, this will also lead to a more complex structure of the algorithm for estimating potato crop mass parameters.

6. Conclusions

A classical methodology for assessing quantitative parameters of agricultural crop biomass is being developed, based on the use of mathematical models. In this paper, this development is extended to assessing the parameters of root crop biomass for conditions when the root crop biomass is inaccessible to Earth remote sensing (ERS) tools, and the number of ERS channels used is less than the number of parameters being assessed.

The main provisions for the development of the methodology are:

- the use of three mathematical models, the first of which is a dynamic model of root crop biomass parameters, reflecting the relationship of the aboveground part of the biomass with the mass of root crops, as well as the dependence of both components on the parameters of the soil environment and meteorological factors;

- the second is a model of the relationship of ERS parameters, reflecting the relationship of the reflectance parameters in the red and near infrared optical range with the parameters of the aboveground part of the root crop (ERS model);

- the third is a model of the dynamics of soil environment parameters, reflecting the removal of nutrients and moisture by the biomass of the root crop;

- formulation of the requirement for observability of the dynamic model for the selected remote sensing channels, as a possibility of assessing all components of the root crop biomass;

- ensuring stability and reducing errors in assessing the parameters of the root crop biomass and soil environment parameters, the assessment algorithm is closed through a remote sensing model with real remote sensing data.

The proposed methodology and software for its practical use does not require special training of personnel and can be applied to other root crops, such as carrots and potatoes.

Author Contributions

Conceptualization, I.M.M.; methodology, I.M.M.; software, V.N.T.; validation, I.M.M. and V.N.T.; formal analysis, I.M.M.; writing—original draft preparation, I.M.M.; supervision, I.M.M.; All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The experimental data and testing results are in Department 140 of the Agrophysical Research Institute of the Russian Academy of Sciences (ilya.mihailenko@yandex.ru).

Conflict of Interest

The authors declare no conflict of interest.

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