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RESEARCH ARTICLE

Impact of Earth Radiation Pressure Physical Analytical Model on Satellite Laser Ranging Orbit Determination

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Abstract: SLR (Satellite Laser Ranging) is a kind of important space geodesy technique for the establishment of a Terrestrial Reference Frame (TRF) and determination of EOP (Earth Orientation Parameters). It determines the origin and scale factor of TRF. The accuracy of future TRF is 1 mm. This requires improving the SLR data processing accuracy and importing higher accuracy SLR satellite data. The Earth Radiation Pressure (ERP) is an important perturbation force for SLR satellites. The traditional Earth radiation pressure model for SLR satellites is the simple Cannon Ball model. This paper establishes the Box-Wing physical Earth radiation pressure model for SLR satellites and takes Lageos-1 as an example to evaluate the physical analytical model. The Lageos-1 is divided into two blocks: metal shell and corner reflectors. The area and optical characteristics of each block are analyzed according to the requirements of three kinds of Earth albedo and emissivity models of point source, experience and grid models. The results show that after importing the physical analytical Earth radiation pressure model, the empirical force acceleration in the T direction is significantly reduced and the orbit overlap arc precision is about 3 mm smaller than that of the original model. The orbit prediction results show that the prediction accuracy of the new Earth radiation pressure model has generally improved significantly. The maximum improvement percentage of the physical analytical model is 12%, 16%, 28% and 25% respectively in the prediction arc length of 1 day, 3 days, 5 days and 7 days. The physical grid model performs the best with the increase of prediction arc length.

Keywords: Satellite laser ranging; Earth radiation pressure; Precision orbit determination; Physical analytical model

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1. Introduction

The solar radiation received by the Earth can be released in two different ways: optical radiation and infrared radiation. Through these two kinds of radiation, the thermal equilibrium of the Earth can be maintained [1,2]. The Earth's radiation perturbation is normally called the Earth Radiation Pressure (ERP). The Earth's radiation pressure consists of the Earth's albedo radiation pressure caused by optical radiation and the infrared radiation pressure caused by infrared radiation. The Earth's albedo radiation pressure generally depends on the sunlight reflected from the ground. When the satellite is in the shadow of the Earth, this item is zero. The infrared radiation pressure results from the heat radiation of the Earth itself and mainly depends on the temperature of the Earth's surface. It has a strong correlation with latitude and season and has almost nothing to do with the position of the satellite. The Earth's radiation pressure is mainly concentrated in the radial direction of the satellite, which is about 100 times larger than the tangential and normal directions. For SLR satellites, the orbit height is usually thousands of kilometers and the impact of the Earth's radiation pressure on its orbit is greater than that of GNSS satellites with tens of thousands of kilometers. Therefore, ERP should be considered for most SLR satellites' POD (Precision Orbit Determination) and other applications. At present the radial impact of the Earth's radiation pressure on GNSS orbit determination can reach the order of centimeter. IGS (International GNSS Service) has proposed to introduce the Earth's radiation pressure into the precision orbit determination of navigation satellites [3]. So, the Earth's radiation pressure must be considered in regular data processing and centimeter even or millimeter order POD of SLR LEO (Low Earth Orbit) and MEO (Middle Earth Orbit). The ERP becomes small with the orbital altitude augment of satellites.

The Earth's optical radiation and infrared radiation can be described by the Earth's albedo A_l and emissivity E_m, respectively. They are affected by many complex factors such as oceans, land and cloud changes. In practical applications, albedo and emissivity could be fitted or modeled numerically by the observed satellite data such as the simplest point source model or the empirical formula derived by CSR (Center for Space Research of the University of Texas at Austin). Because they are difficult to be derived by theoretical formulas NASA uses the data provided by CERES (Clouds and the Earth Radiant Energy System) to calculate the global grid radiation data and obtain the grid albedo and emissivity of the Earth. Based on Box-Wing theory and Fliegel's work [4], Rodriguez-Solano established the point and numerical grid Earth Radiation pressure models for GPS satellites by CERES data respectively [3,5–7]. Zhao applied this method to BDS satellites and established the physical analytical model of the Earth's radiation pressure for the BDS satellites. The orbit determination results showed that the IGSO/MEO satellite's orbit accuracy has a 2–10 mm improvement after the introduction of the Earth radiation pressure model [8].

Although SLR satellite's Earth radiation pressure research started early, the Earth radiation pressure model adopted by the Analysis Centers of the ILRS (International Laser Ranging Service) is still dominated by the spherical model (Cannon Ball) because conventional geodesic satellites are almost uniform spheres [9–12]. This Cannon Ball model is simple, and only the mass and cross-sectional area of the satellite are used to model the Earth's radiation pressure. However, the actual SLR satellite is not an ideal uniform symmetrical sphere including Lageos-1/2. Its surface is composed of different materials such as metal shells and corner reflectors. Different parts have different optical characteristics. The accuracy of the Earth's radiation pressure model has an important impact on the Lageos-1/2 satellite. Its impact on the long half axis of the orbit and the scale factor is about 1.5 mm and 0.07 ppb respectively [10,11,13,14]. Therefore, with the improvement of SLR observation accuracy, the goal realization for future 1 millimeter level space-time benchmarks and higher precision SLR orbit determination, it is very necessary to establish a set of high-precision SLR Earth radiation pressure physical analytical models to improve the SLR data processing accuracy. This is useful to better leverage the important role of SLR in determining time-space reference frames and satellite orbits.

This article takes the improvement of the high-precision SLR Earth radiation pressure physical analytic model as the research background, different Earth albedo and emissivity models are introduced. Combined with these models, high-precision Earth radiation pressure physical analytic models are established for SLR satellites and evaluated.

2. Earth Albedo and Emissivity Models

If the incident flow intensity of direct solar radiation on the Earth is I_S, the optical radiation intensity of the light reflected by the Earth is I_{refl}, the infrared radiation intensity of the Earth is I_{emit}, then the calculation formulas for the earth albedo and emissivity are given respectively by equation (1).

$$\alpha = \frac{I_{refl}}{I_s}$$
$$\varepsilon = \frac{I_{emit}}{I_o} \tag{1}$$

At present, there are three kinds of main models for the Earth's albedo and emissivity: constant model, empirical model, and numerical grid model. The constant model takes the global Earth albedo and emissivity as constants (α = 0.3, ε = 0.7), equivalent to a constant radiation amount of 459 W/m^2 on the Earth's surface and the average infrared radiation intensity 341.8 W/m^2 on the Earth's surface [4,15]. The empirical model is currently mainly an analytical expression based on satellite observation fitting provided by the Center for Space Research (CSR) in the United States. This model fits the Earth's albedo and emissivity according to the periodic formula of latitude and time. The specific calculation formula is given by equation (2) [2,8].

$$\alpha = 0.34 + 0.1\cos\left[\frac{2\pi}{365.25}(t - t_0)\right]\sin\varphi + 0.29(\frac{3}{2}\sin^2\varphi - \frac{1}{2})$$
$$\varepsilon = 0.68 - 0.07\cos\left[\frac{2\pi}{365.25}(t - t_0)\right]\sin\varphi - 0.18(\frac{3}{2}\sin^2\varphi - \frac{1}{2}) \tag{2}$$

In the equation, φ is the latitude, t and t_0 are the calculation time and the initial reference time of the periodic term.

The most accurate model for Earth's albedo and emissivity is the numerical grid model, which utilizes the global top of atmosphere (TOA) solar radiation flow, the Earth's shortwave reflected radiation flow and longwave infrared radiation flow monitoring products provided by NASA CERES observation program. It can be used for calculating the Earth's albedo and emissivity at a resolution of 1° × 1°. CERES grid data has been widely used in the calculation of Earth's radiation pressure. Figures 1 and 2 respectively show the global distribution and difference between the empirical model and numerical grid model of Earth's albedo and emissivity in March 2019. From these figures, it can be seen that the empirical mode cannot accurately depict the status of complex areas because it only considers latitude changes, while the numerical grid model uses measured data and better shows the detailed changes in complex areas.

3. Earth Radiation Pressure Models

At present, the Earth radiation pressure (ERP) models mainly include the point source ERP model, empirical ERP model, sphere ERP model, numerical grid ERP model, etc. Different models have different usage conditions, simplification levels and accuracy. They can be selected based on data processing conditions and accuracy requirements.

For satellites with complex shapes, the reflectivity and specular reflectance of each area element meet the following requirements [8,15]:

$$v_i\mu_i + (1 - \mu_i)v_i + (1 - v_i) = 1 \tag{3}$$

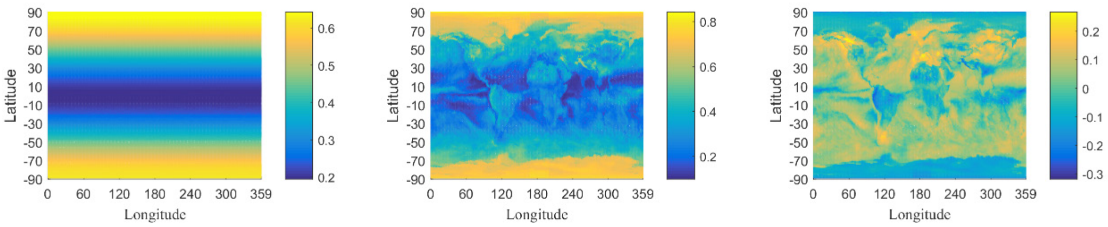

Figure 1. The experience Earth albedo (left) and grid Earth albedo (middle) in March 2019 and their difference (right).

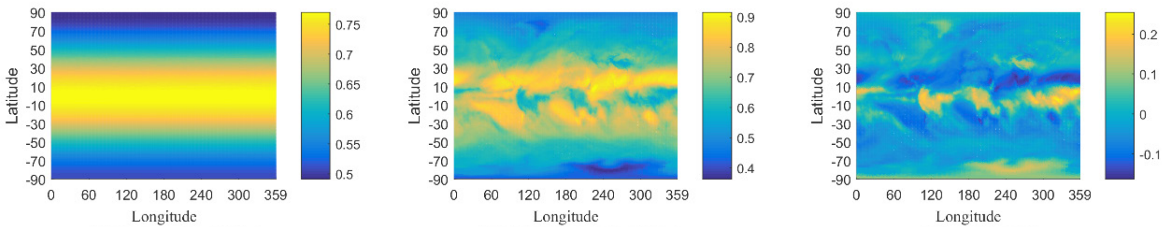

Figure 2. The experience Earth emissivity (left) and grid Earth emissivity (middle) in March 2019 and their difference (right).

where, ν_i is the reflectivity of the area element; μ_i is the specular reflectance of the area element.

The albedo radiation pressure and infrared radiation pressure of each surface element are:

$$\vec{A}_{refl} = -\lambda \frac{E_{refl}}{mc} \sum_{i=1}^{n} A_i \cos\theta_i \left\{ 2\nu_i \left[\mu_i \cos\theta_i + \frac{1-\mu_i}{3} \right] \hat{\boldsymbol{n}}_i + (1-\mu_i \nu_i) \boldsymbol{p}_i \right\}$$
$$\vec{A}_{emit} = -\lambda \frac{E_{emit}}{mc} \sum_{i=1}^{n} A_i \cos\theta_i \left\{ 2\nu_i \left[\mu_i \cos\theta_i + \frac{1-\mu_i}{3} \right] \hat{\boldsymbol{n}}_i + (1-\mu_i \nu_i) \boldsymbol{p}_i \right\} \tag{4}$$

where, $\vec{A}_{refl}$ is the Earth's albedo radiation pressure, in unit m/s^2; $\vec{A}_{emit}$ is the Earth's infrared radiation pressure; E_{refl} and E_{emit} are the Earth reflected radiation flux and emission radiation flux at the satellite's location, in unit w/m^2; λ is the shadow factor of satellite, m is the mass of satellite in unit kg; c is the speed of light, 299792458 m/s; A_i is the area of the surface element i, in unit m^2; θ_i is the angle between the normal vector of the area element and the direction from the satellite to the sun, in unit rad; $\hat{\boldsymbol{n}}_i$ is the normal vector of the surface element; $\hat{\boldsymbol{p}}_i$ is the direction vector from the satellite to the sun. The force vector sum of each surface element is calculated to obtain the Earth's albedo radiation pressure and infrared perturbation acceleration, respectively.

3.1 Point Source ERP Model

The point source ERP model considers the Earth as an ideal point, with constant Earth albedo and emissivity. By equation (4) the Earth's albedo and infrared radiation pressure of each surface of satellites can be calculated and their vector sum is the Earth's radiation pressure.

3.2 Empirical ERP Model

Similar to the point source model, the Earth albedo and emissivity used are no longer constants, but are calculated by its empirical equation (2). Then, the Earth's radiation pressure can be calculated by equation (4).

3.3 Cannon Ball ERP Model

In order to minimize the impact of non-conservative forces as much as possible, reduce the complexity of satellites and ensure satellite stability, SLR geodynamic satellites are usually designed as spherical and use high-density metal cores to maximize satellite mass to reduce the impact of Earth radiation pressure. For SLR spherical satellites, both the traditional Earth radiation pressure model and the current model ILRS analysis center adopted is the spherical model, which treats the satellite as a completely uniform surface sphere. The calculation formula for the spherical ERP model is:

$$\vec{a}_{eradp1} = \rho_{SR} \left(\frac{A_U}{r_s} \right)^2 \left(\frac{A}{m} \right) \cdot \eta_l \cdot \vec{r}_p \cdot \left(\frac{(f \cdot \alpha \cdot \cos\theta_s + \varepsilon)\cos\gamma}{\pi \left| \vec{r}_p \right|^2} \right) \tag{5}$$

where, ρ_{SR} is the constant of the solar pressure near the Earth; A and m is the cross-sectional area and mass of the satellite, respectively; A_U is an astronomical unit; r_s is the distance from the sun to the Earth; η_l is the Earth radiation pressure coefficient; $\vec{r}_p$ is the vector from the ground radiation point to the satellite; θ_s is the incidence angle of solar radiation at the Earth radiation point; γ is the angle between the normal vector of radiation point and $\vec{r}_p$. This model is simple and conveniently be calculated only by utilizing the cross-sectional area and mass of the satellite without the need for more detailed information.

3.4 ERP Physical Analytical Model

Fliegel et al. [4] used two optical characteristic parameters: reflectance (v, 0-1) and specular reflectance (μ, 0-1) to describe the acceleration generated by the interaction between solar, Earth radiation and satellites. In this case, the absorption rate of the satellite is $1-\nu$, the specular reflection part is $\nu\mu$, and the diffuse reflection part is $\nu(1-\mu)$. According to Box Wing theory [6,16], the acceleration of the Earth's radiation pressure could be calculated by:

$$\vec{a}_{eradp} = -\frac{E_0 A}{mc} \cos\theta \left[(1-\nu\mu)\vec{e}_D + \frac{2}{3}(1-\nu\mu+3\nu\mu\cos\theta)\vec{e}_N \right] \tag{6}$$

where, $\vec{e}_D$ is the direction vector from the sun to the satellite; e_N is the normal vector of the satellite surface, $\cos\theta = \vec{e}_D \cdot \vec{e}_N$; E_0 is the Earth's radiation intensity at the satellite location. In actual calculations, the Earth surface needs to be integrated to obtain this value. The calculation formula is given by:

$$\begin{aligned} E_0 &= E_{0\hat{r}} + E_{0r_\perp} \\ E_{0\hat{r}} &= \int dE_{refl} \cdot \hat{r} + \int dE_{emit} \cdot r \\ E_{0\hat{r}_\perp} &= \int dE_{refl} \cdot \hat{r}_\perp \end{aligned} \tag{7}$$

$E_{0\hat{r}}$ and $E_{0\hat{r}_\perp}$ are the radial and lateral components of Earth's radiation. The infrared radiation only includes the radial components. The calculation formula for

dE_{refl} and dE_{emit} are given by:

$$dE_{refl} = \begin{cases} \dfrac{\alpha}{\pi d^2} \cos\theta \cos\gamma E_S dA\hat{e} \; (\cos\theta \geq 0 \; \& \cos\gamma \geq 0) \\ 0 \; (\text{other condition}) \end{cases}$$

$$dE_{emit} = \begin{cases} \dfrac{\varepsilon}{\pi d^2} \cos\gamma E_S dA\hat{e} \; (\cos\gamma \geq 0) \\ 0 \; (\text{other condition}) \end{cases} \tag{8}$$

where, d is the distance from the ground radiation integration grid point to the satellite; E_S is the solar constant; $\hat{e}$ is the direction vector from the radiation grid to the satellite; dA is the grid area; other parameters are consistent with the previous text.

4. SLR ERP Physical Analytical Model Establishment and Test Schemes

By the theory of the above ERP models, the physical analytical ERP model was constructed taking the Lageos-1 satellite as an example. Table 1 provides the basic information and optical characteristics of the satellite. By these parameters, a physical analytical model for the Earth radiation pressure of the SLR satellite was established and compared with other models, especially the spherical model widely used in SLR data processing.

Table 1. Information for Lageos-1 satellite.

Basic Information	Value
Height/km	5860
Mass/kg	411.0
Diameter/m	0.60
Number of reflector (CCR)	426
CCR diameter/m	0.038

In order to construct the physical analytical model of the Earth's radiation pressure, it is necessary to obtain the area and optical characteristics of each component of the satellite. The Lageos-1 satellite is mainly divided into two parts: an aluminum alloy shell and corner reflectors. Due to the early launch of the satellite, it is difficult to obtain accurate information on the surface metal of the satellite. However, it is currently known that in the early stages of the Lageos-1/2 satellite, due to the need for optical observation, the roughness of the satellite's metal surface was very low. Therefore, its reflectivity and specular reflectivity were set to 0.9 and 0.8, respectively. The reflectance and specular reflectance of corner reflectors due to their strong reflection characteristics are considered as 1.0 here. The surface area of the SLR satellite remains constant regardless of time and location because of its spherical shape and almost uniform reflector distribution. It means that the area radiated by the ground does not change without considering complex attitude changes, as shown in Figure 3. Calculate the geocentric angle of the radiation range based on the height of the satellite by:

$$\theta = \text{acos}\,[r/(r + h)] \tag{9}$$

where, θ is the geocentric angle of the radiation range; r is the radius of the Earth; h is the height h of the satellite. Then, the radiation area of the satellite and the Earth's radiation pressure could be calculated.

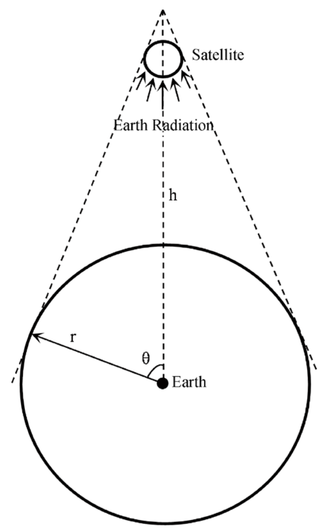

Figure 3. Schematic of satellite receiving Earth radiation.

Moreover, it is worth mentioning the Lageos satellite is covered with 426 cube corner reflectors with all but four of these reflectors made of fused silica glass. This doesn't have any effect on the POD results because the reflectance and specular reflectance of the four other material corner reflectors due to their strong reflection characteristics are also considered as 1.0.

In order to verify the accuracy of the SLR ERP physical analytical model, the Lageos-1 satellite's ILRS global observation data from 2019 to October 2020 were selected for orbit determination testing. The physical analytical modeling information is shown in Table 2. In order to verify the impact of different Earth albedo and emissivity models, the newly established SLR ERP physical analytical model was compared with three kinds of the Earth's albedo and emissivity models and the original spherical model. The orbit determination models and calculation strategy are shown in Table 3 [17,18] and the different ERP test schemes are shown in Table 4.

Table 2. Physical analytical modeling information for Lageos-1 satellite.

Term	Aluminum Alloy Shell	CCR
Total area/m^2	0.6469	0.4831
Illuminated area/m^2	0.415	0.310
Reflectance	0.9	1.0
Specular reflectance	0.8	1.0

5. Result and Analysis

To comprehensively evaluate the accuracy and characteristics of the newly established SLR ERP physical analytical model, this article conducted a comprehensive analysis from the aspects of acceleration, empirical force changes, orbit overlap arc accuracy and orbit prediction accuracy in ERP calculation.

5.1 Acceleration Analysis

Figure 4 shows the space distribution of Earth radiation pressure acceleration of the Lageos-1 satellite in the R (Radial)/T (Tangential)/N (Normal) direction as a function of solar altitude angle β_0 and the latitude angle △ u relative to the sun. It shows that the perturbation acceleration is mainly concentrated in the radial direction, which is about 100 times that of the other two directions. The acceleration in the R direction is on the order of 10^{-10}–10^{-9}, and T and N are on the order of 10^{-12}–10^{-11}. When β_0 = △ u = 0, the maximum is reached.

Figure 5 shows the variation of acceleration over time for different Earth radiation pressure models in the R/T/N direction over a period of 3 days, the horizontal axis is MJD (Modified Julian Dates). Due to the point model treating the Earth as a point, the acceleration is only distributed radially along the line connecting the point and the satellite. The difference in radial acceleration between the original model and the physical analytical model is significant, several times that of the original model. The main reason is that in the original spherical model the calculation of perturbation acceleration only considers the cross-sectional area and treats the satellite as a completely uniform sphere. In the physical analytical model, it is considered that the radiation to the satellite is generated by the entire hemisphere of the Earth, with a larger area compared to the cross-sectional area. And also considering the differences in optical characteristics of different components of the satellite, the larger area and strong reflectivity of satellite components result in a larger calculated radial radiation acceleration.

Table 3. Orbit determination models and calculation strategy for Lageos-1 satellite.

Term	Model and Strategy
POD arc length	7 days, 150 s steps
Toposphreic model	Mendes-Pavlis
Tidel model	Earth solid tide, ocean tide and pole tide
Relativity	Shapiro time delay
COM	Site dependent (0.245–0.251 m)
Gravity model	EGM2008 (30 × 30)
Three body perturbation	JPL DE420
Atmospherical drag perturbation	Drag-like force
Solid and ocean tide perturbation	FES2004/IERS2010
Earth radiation pressure perturbation	Different test models
Reference frame	SLRF2014
EOP	IERS EOP C04

Table 4. The different Earth radiation pressure models test schemes.

Scheme	Model	Identification
1	Cannon ball + empirical albedo and emissivity	Origin model
2	Physical analytical model + point source albedo and emissivity	Physical point source model
3	Physical analytical model + empirical albedo and emissivity	Physical empirical model
4	Physical analytical model + grid albedo and emissivity	Physical grid model

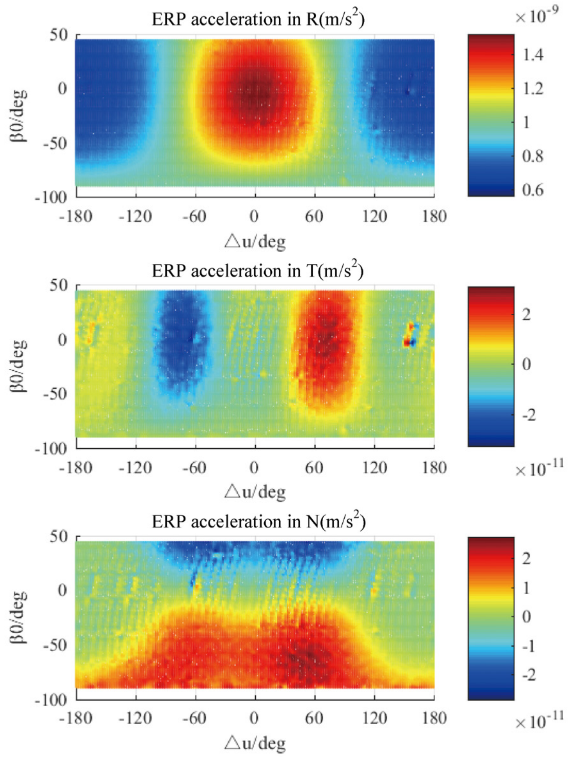

Figure 4. Earth radiation acceleration in R/T/N direction with the variation of the altitude angle β_0 of the sun and argument of latitude w.r.t. the Sun △ u.

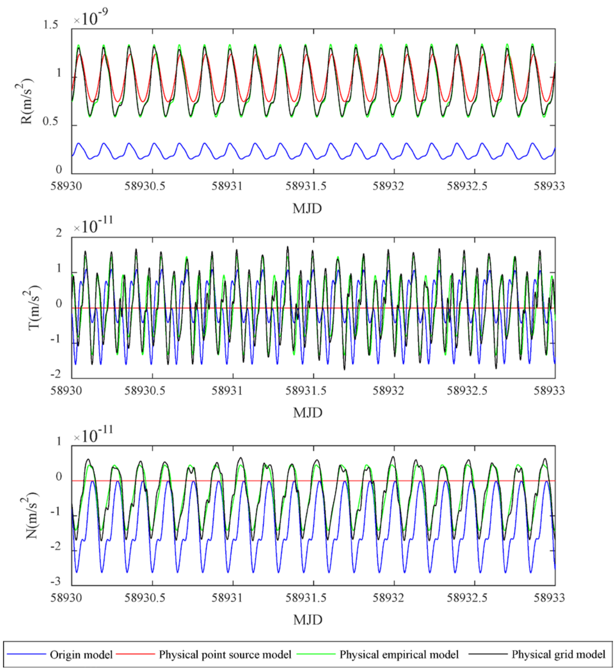

Figure 5. Acceleration changes in the R/T/N direction of different models within 3 days.

5.2 Empirical Force Changes

In SLR orbit determination, to absorb the errors caused by unmodeled and inaccurate models, a set of empirical forces is usually estimated. The empirical forces are expressed as a set of periodic functions related to the satellite's latitude angle u, as shown in equation (10).

$$\vec{a}_{RTN} = \begin{bmatrix} (R_C \cos u + R_S \sin u)\hat{u}_R \\ (T_C \cos u + T_S \sin u)\hat{u}_T \\ (N_C \cos u + N_S \sin u)\hat{u}_N \end{bmatrix} \tag{10}$$

where, R_C, R_S, T_C, T_S, N_C, and N_S, are the coefficients to be estimated. In SLR data processing only the empirical forces in the T and N directions are estimated in general. Figure 6 shows the changes in sin (u) and cos (u) over 3 days, and Figure 7 shows the changes in estimated values of empirical force acceleration in the T and N directions within three days (7–10 Nov. 2019).

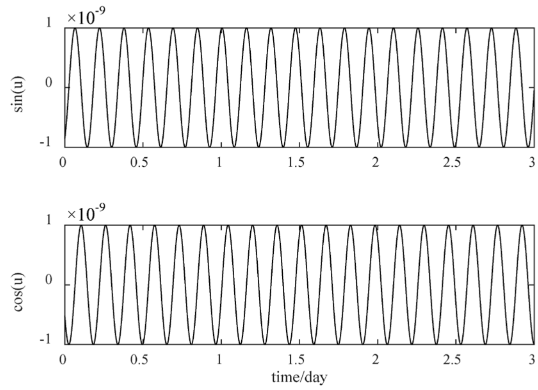

Figure 6. Lageos-1 sin (u) and cos (u) change in 3 days.

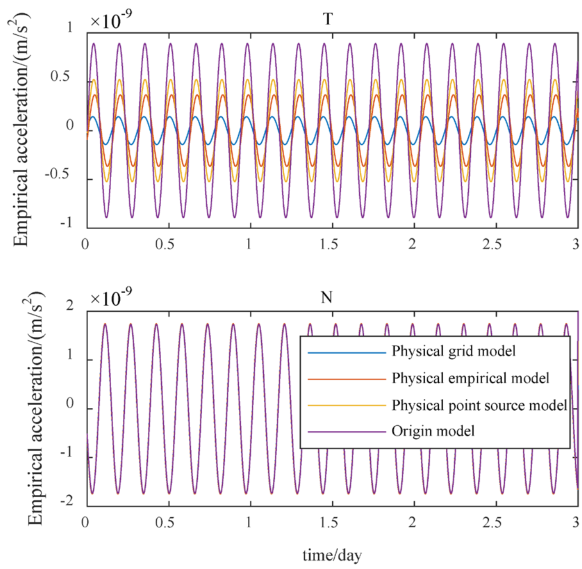

Figure 7. Empirical acceleration change of T (up) and N (bottom) direction in 3 days for different ERP models.

The empirical force acceleration of point source, empirical, and grid physical models has decreased by 41%, 58%, and 83% in the T direction compared to the original model, respectively. But there is basically no change in the N direction. This indicates that the accuracy of the new model significantly reduces the force used to absorb inaccurate modeling.

5.3 Orbit Overlap Arc Accuracy Analysis

Orbit overlap arc accuracy is a method of characterizing the accuracy of coincidence in precise orbit determination (POD). Observation data from overlapping observation arcs with a sliding window are used for orbit determination and the results of two PODs are compared. The difference in orbital positions during the overlapping arc can be used as an indicator of the accuracy of the ERP models. This article uses a 7-day sliding window for orbit determination, with a 2-day overlap between two PODs. The orbit overlap accuracy of all arcs is calculated, Figure 8 shows the overlap accuracy of different model orbits. The differences between different physical analytical models (schemes 2–4) are relatively small. But compared to the original model, they all have an accuracy improvement of about 3 mm.

5.4 Orbit Prediction Accuracy Analysis

Another good method for verifying the accuracy of ERP models is orbit prediction accuracy evaluation, in which the perturbation model error is amplified over time to better evaluate the model's performance. The force model for orbit prediction is consistent

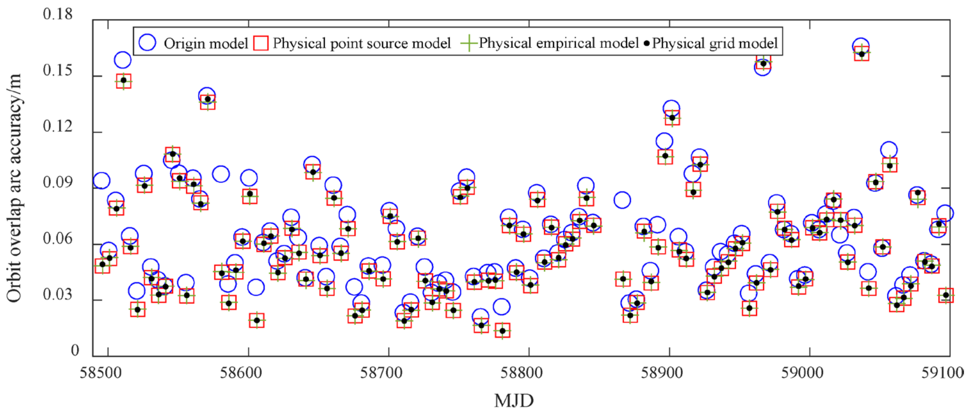

Figure 8. The difference in orbit overlap arc accuracy between the original model and different physical models.

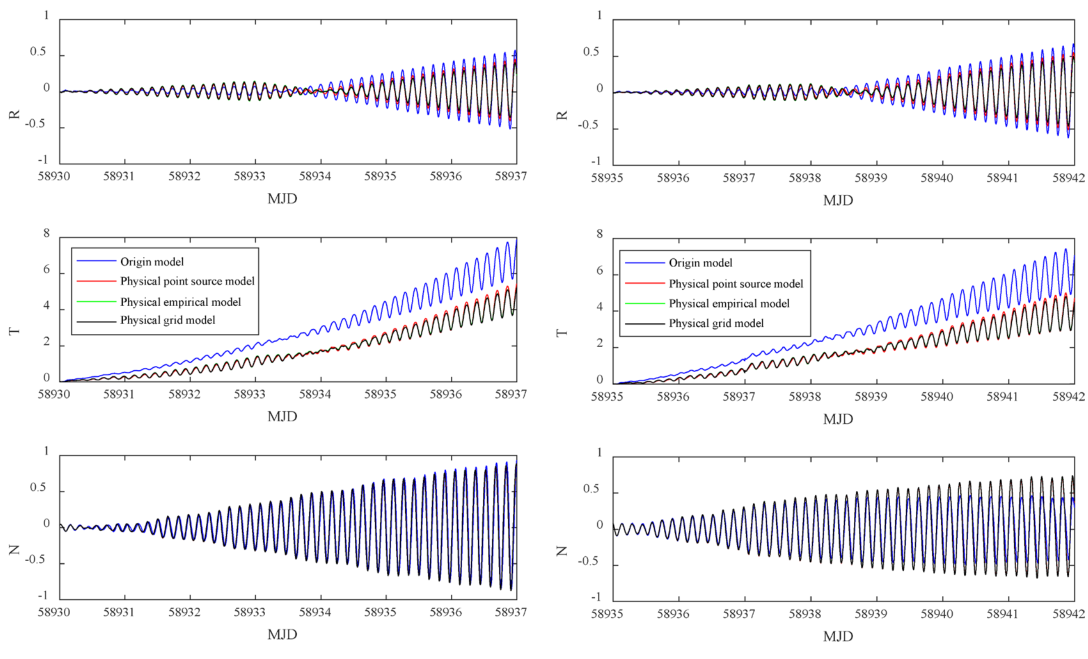

Figure 9. R/T/N difference of 7 days predicted orbit w.r.t precise orbit (two 7-day prediction results).

with Table 3. Figure 9 shows the comparison results of two 7-day prediction arcs in R/T/N directions. Compared with the original model, the new physical model (schemes 2–4) has improved the accuracy of 7-day orbit prediction accuracy. The orbit prediction accuracy improvement is mainly concentrated in the tangential direction and the maximum improvement can reach several meters. The grid model (scheme 4) performs the best. Because the biggest change of Earth radiation pressure is concentrated in the radial direction as shown in Figure 5, the radial acceleration directly causes changes in the radial height of the satellite and leads to changes in satellite velocity. The satellite velocity is in the tangential direction, and the corresponding changes in orbit prediction accuracy are directly reflected in the tangential direction. This is similar to the calculation results of BDS and GPS Earth radiation pressure [19,20].

In order to obtain the long-term orbit prediction accuracy of the physical analytical model, the orbit determination and prediction from 2019 to October 2020 was done with a prediction arc length of 7 days. The RMS and maximum of each arc's orbit error were calculated, as shown in Figure 10. From Figure 10, it can be seen that the new physical model has a generally significant improvement in orbit prediction accuracy compared to the original model and there are certain differences between physical analytical models with different albedos and emissivity. But the difference compared to the original model is still much smaller.

Figure 11 and Table 5 provide statistics on the orbital error RMS and maximum error values for 1-day, 3-day, 5-day and 7-day prediction arc length, respectively. Physical analytical models with different prediction arc lengths outperform the origin model. The physical analytical grid model performs the best. The prediction accuracy improvements for 1-day, 3-day, 5-day and 7-day predictions were 0.09 m, 0.41 m, 1.28 m and 2.01 m, respectively, with an improvement percentage of 12%, 16%, 28%, and 25%. This demonstrates the correctness and reliability of the physical analytical models. By combining the different model results mentioned above, the Earth's albedo and emissivity models from the numerical grid have the highest accuracy in the Earth's radiation pressure calculation due to their precise data sources.

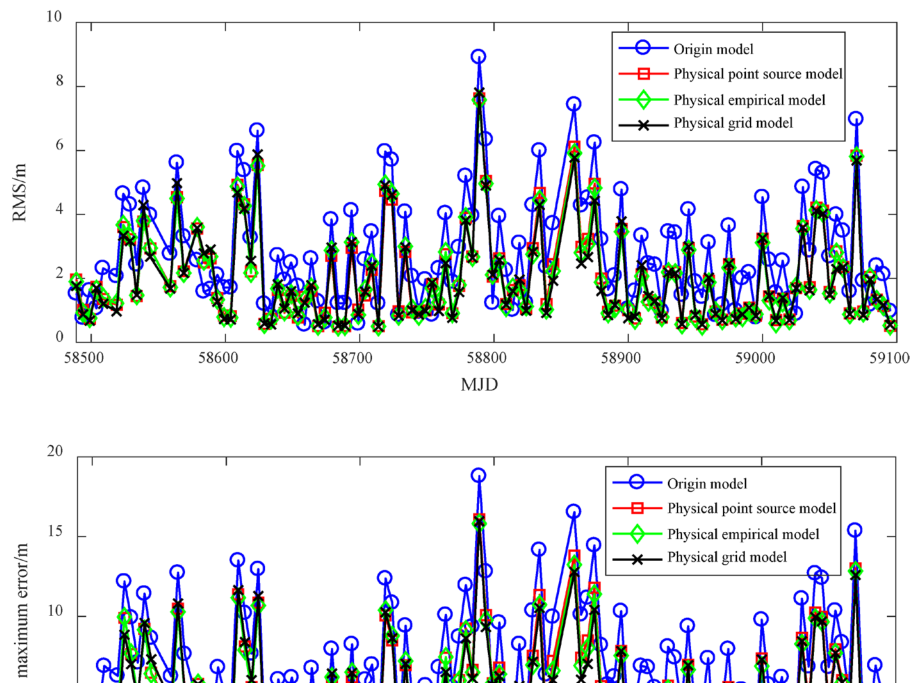

Figure 10. SLR orbit prediction error RMS and maximum error statistics from different ERP models for about 2 years.

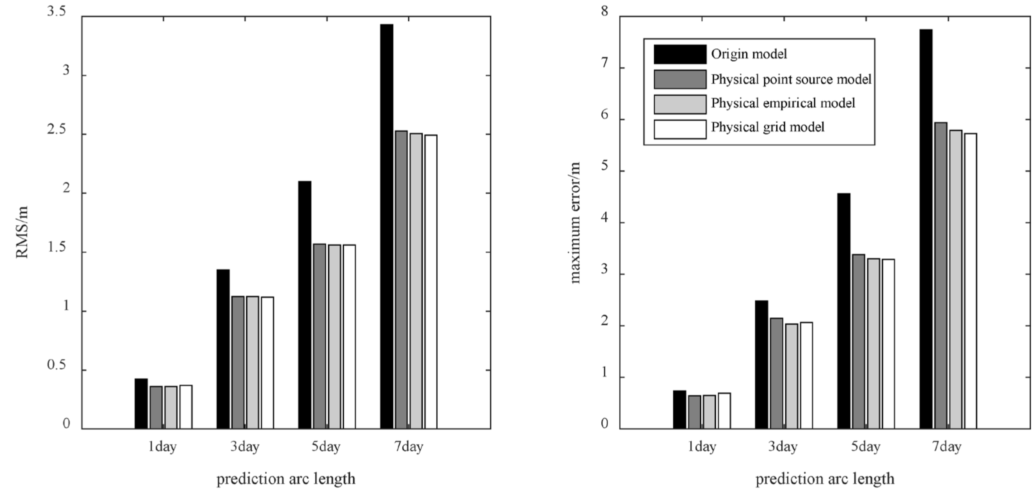

Figure 11. Orbit prediction RMS and maximum errors for different prediction arc lengths.

Table 5. Orbit prediction RMS and maximum error statistics for different prediction arc lengths.

	Scheme 1	Scheme 2	Scheme 3	Scheme 4
1 D RMS/m	0.4223	0.3607	0.3607	0.3690
3 D RMS/m	1.3491	1.1235	1.1241	1.1176
5 D RMS/m	2.0984	1.5690	1.5613	1.5610
7 D RMS/m	3.4325	2.5288	2.5069	2.4929
1 D Maximum/m	0.7339	0.6436	0.6514	0.6965
3 D Maximum/m	2.4798	2.1431	2.0377	2.0620
5 D Maximum/m	4.5627	3.3847	3.3042	3.2879
7 D Maximum/m	7.7373	5.9411	5.7910	5.7292

6. Conclusions

This article applies the modeling approach of the box-wing solar pressure model to establish the SLR Earth radiation pressure models and takes the Lageos-1 satellite as an example to evaluate the different ERP models. Combining different Earth albedo and emissivity models, three SLR Earth radiation pressure physical analytical models were developed by taking into account the actual structure and optical characteristics of the SLR satellite. A comprehensive comparison and accuracy analysis was conducted with the traditional SLR Earth radiation pressure ball model. The results show that the introduction of the new SLR Earth radiation pressure physical analytical model significantly reduces the empirical force in the T-direction, improves the orbit overlap accuracy by about 3 mm, and improves the accuracy of 1-day, 3-day, 5-day, and 7-day orbit prediction by 12%, 16%, 28%, and 25%, respectively. This is of great significance for high-precision SLR orbit determination and application research, especially for establishing spatiotemporal benchmarks with 1mm accuracy requirements in the future. Moreover, the method of the new SLR Earth radiation pressure physical analytical model is also suitable for other non-spherical satellites with nonhomogeneous optical characteristics materials although the Lageos-1 satellite is taken as an example to evaluate the different ERP models. This will be very important for the POD accuracy improvement of other satellites such as Starlette, Stella, AJISAI, HY-2A, Jason-2/3, CHAMP, GRACE, GOCE and so on.

Author Contributions

XW proposed the study, supervised the progress of the study, gave advice for issues, and revised the manuscript. HY realized the model, analyzed the result, and wrote the manuscript. YL contributed to the refinement of software. All authors commented on the manuscript draft and approved the submission. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The SLR datasets are available from https://cddis.nasa.gov/. The Earth albedo and emissivity grid data are from https://ceres.larc.nasa.gov/data/.

Conflict of Interest

The authors declare that they have no conflict of interest.

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