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#### RESEARCH ARTICLE

# Revisiting Recent Amplitude and Phase Variations of the Chandler Wobble and Free Core Nutation

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## 1. Introduction

The rotation of the Earth is a very complex process that consists of many free and forced modes. Analysis of the variations in the Earth's rotation plays an important role in our understanding of the physical processes in the Earth's surface, interior, atmosphere, and hydrosphere. Three main constituents of the Earth's rotation are movement of the rotation axis in terrestrial and celestial reference frames, and rotation around the rotation axis. In this paper,

#### ABSTRACT

The paper is devoted to the analysis of two components of the Earth's rotation, Chandler wobble (CW) and free core nutation (FCN). They are oscillations with near-constant periods but variable amplitude and phase. The variations of the amplitude and phase of the CW and FCN have already been considered in the literature, and both showed similar behavior such as a recent significant decrease of the amplitude and large phase change. However, the CW and FCN amplitude and phase variations are, to a large extent, predicted for the current epochs, and their today's variations need regular updates with obtaining new observations. In this work, the CW and FCN parameters have been re-computed using the latest data and compared with the data published earlier. It was found that the currently obtained amplitude and phase variations generally agreed with the data published earlier. The main difference is that the epochs of the current minimum of amplitude and phase jump or both CW and FCN happened somewhat later than was predicted in previous publications. The delay is about two years for the CW relative to the prediction made in 2010 and about one year for the FCN with respect to the prediction made in 2022.

some features of the first two processes will be discussed.

The movement of the Earth's rotation axis relative to the Earth's surface manifests itself as Polar motion (PM) and is observed through the coordinates of the terrestrial Pole  $X_p$  and  $Y_p$  in the conventional terrestrial reference frame <sup>[1]</sup>. The Chandler wobble (CW) discovered by Seth Carlo Chandler in 1891 <sup>[2,3]</sup> is one of the main and most complicated components of the Earth's rotation, and numerous papers were devoted to investigation of the CW

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variations, such as changes in the CW amplitude and phase. A brief overview of these studies is given [4-10] and papers cited therein. The main result [4] was detecting, for the first time, a large jump in the CW phase in the 2000s and a simultaneous deep minimum of the CW amplitude.

Free core nutation (FCN) is a component of the motion of the Earth's rotation axis in the conventional celestial reference frame <sup>[1]</sup>. The FCN is a free Earth's rotational mode caused by the misalignment of the rotational axis of the Earth's mantle and the rotational axis of the outer liquid core <sup>[11]</sup>. It is observed through the coordinates of the celestial Pole *dX* and *dY* in the conventional celestial reference frame <sup>[1]</sup>. Like CW, the FCN is an oscillation with highly variable amplitude and phase that was investigated in many studies <sup>[12-17]</sup> and papers cited therein. In a recent paper <sup>[18]</sup>, a new minimum of the FCN amplitude and simultaneous large jump in the FCN phase was preliminary detected at the epoch around 2022.

It should be noted that the CW and FCN amplitude and phase variations [4,18] were obtained from analysis of the series of Earth orientation parameters (EOP) ended near the epochs of the investigated minima of the amplitude and phase jumps. Besides, both series used in these works are smoothed, the CW series to a lesser extent, and the FCN series to a larger extent. Under these circumstances, today's behavior of the CW and FCN could not be accurately predicted when these papers were published. Therefore, it is very desirable to revise the results [4,18] using the latest available observations, which is the primary goal of this study.

The paper is organized as follows. Section 2 describes the analysis of the CW variations, and Section 3 is devoted to the analysis of the FCN variations. Section 4 sums up the paper.

#### 2. Chandler Wobble

The first step in the CW analysis was to extract the CW signal from the PM time series, removing all the trend, periodic, and quasi-periodic components beyond the CW frequency band. For this study, the CW signal was extracted from the IERS (International Earth Rotation and Reference Systems Service) Pole coordinates time series IERS  $C01^{\oplus}$  using a band-pass digital filter based on the Fourier transform with the window 1.18...1.20 yr centered at the nominal CW period  $P_{CW} = 1.19$  years. The IERS C01 series begins in 1846.0 and extends to the present (April

2023). It is sampled at 0.1 Besselian year (36.524 solar days) for the period 1846-1890 and 0.05 Besselian year (18.262 solar days) after 1890.0. Using this data, a new interpolated EOP series of the CW component of polar motion  $X_{CW}$  and  $Y_{CW}$  with 10 days step was constructed and used for further analysis. This series is shown in Figure 1. Epochs in all the plots in this paper are expressed in Besselian years.

For this work, PM (CW) series started with 1900.0 was used because the data before 1900 are less reliable. The whole IERS C01 series was considered [4], while in this study we are mostly interested in the CW behavior in recent years.

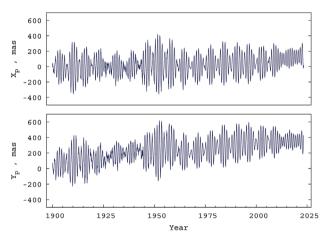


Figure 1. IERS C01 series.

A general CW model can be expressed as follows:

$$X_{CW} = A_c \cos\phi - A_s \sin\phi,$$

$$Y_{CW} = A_c \sin\phi + A_s \cos\phi,$$

$$A_c = -X_{CW} \sin\phi + Y_{CW} \cos\phi,$$

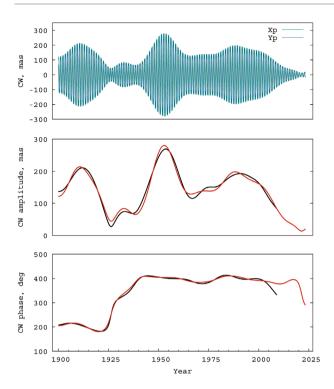
$$A_s = X_{CW} \cos\phi + Y_{CW} \sin\phi,$$
(1)

where CW phase  $\phi = 2\pi/P_{CW}(t-t_0)$ , t is the epoch at which observed  $X_{CW}$  and  $Y_{CW}$  values are given,  $t_0 = J2000.0$ . Each group of Equation (1) corresponds to one epoch given in the CW series. Then the instant CW amplitude and phase at epoch t can be found as:

$$A_{CW} = \sqrt{X_{CW}^2 + Y_{CW}^2} = \sqrt{A_c^2 + A_s^2}$$
  
 $P_{CW} = \arctan \frac{A_c}{A_c}$ . (2)

The results of this analysis are shown in Figure 2. For a better representation of the CW phase variations, the linear trend corresponding to the CW frequency was removed from the CW phase series. The results of the previous analysis [4] are also shown in these plots. Note that the IERS C01 EOP series, which was analyzed [4], ended in December 2008 (epoch 2009.0).

 $<sup>\</sup>textcircled{1}$  https://datacenter.iers.org/data/latestVersion/EOP\_C01\_IAU2000\_1846-now.txt



**Figure 2.** Chandler wobble (CW) series (upper panel), CW amplitude (middle panel), and CW phase minus linear drift (bottom panel). In the amplitude and phase plots, data from the previous publication are shown in black, and data obtained in this work are shown in red.

A well-known large CW phase jump in the 1920s is clearly visible in Figure 2, but we are mostly interested in the comparison of the second large CW phase jump epoch (after 2000.0) detected [4] in this work. One can see that the present analysis revealed an even larger CW phase jump, but its epoch is later than was suggested [4]. One of the reasons may be that the minimum of the CW amplitude was reached after 2010, at the epoch 2021.4 according to this work, which made the determination of the moment of the phase jump made in 2010 not very reliable.

Obtained results for CW variations are in good agreement with the results [9] derived by a similar method of digital filtration of the IERS EOP series but using another filtering technique.

#### 3. Free Core Nutation

The FCN amplitude and phase variations were studied using the ZM3 FCN model in the same way as it was done [18]. First, the ZM2 celestial pole offset (CPO) model was constructed by the Gaussian smoothing and interpolation at daily intervals of the combined EOP series maintained by the International VLBI Service for Geodesy and Astron-

omy<sup>©</sup> (IVS). Figure 3 shows the ZM2 CPO model along with the underlying IVS CPO series.

Then the ZM3 model (FCN series) was evaluated using the following expression:

$$dX = A_c \cos \phi - A_s \sin \phi + X_0,$$
  

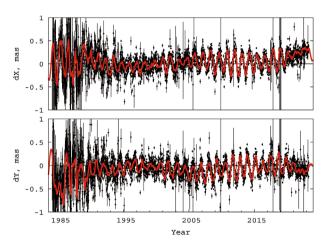
$$dY = A_c \sin \phi + A_c \cos \phi + Y_0,$$
(4)

where  $\phi = 2\pi/P_{FCN}(t-t_0)$ ,  $P_{FCN}$  is the FCN period equal to -430.21 solar days, t is the epoch at which observed dX and dY values are given,  $t_0 = J2000.0$ . The model parameters  $A_c$ ,  $A_s$ ,  $X_0$ , and  $Y_0$  are adjusted by the least square method for running 431-day intervals with one day shift. Each pair of Equation (3) corresponds to one CPO epoch given in the ZM2 series. The model parameters  $A_c$ ,  $A_s$ ,  $X_0$ , and  $Y_0$  were computed at the middle epoch of each 431-day interval.

The FCN contribution to the celestial pole motion at the same epochs is computed by using Equation (3) without the bias terms  $X_0$  and  $Y_0$ .

$$dX_{FCN} = A_c \cos\phi - A_s \sin\phi,$$
  

$$dY_{FCN} = A_c \sin\phi + A_s \cos\phi.$$
(4)



**Figure 3**. ZM2 FCN series (red line) and IVS combined series (black dots with error bars).

Then the instant amplitude  $A_{FCN}$  and phase  $P_{FCN}$  of the FCN signal can be computed as:

$$A_{FCN} = \sqrt{dX_{FCN}^2 + dY_{FCN}^2} = \sqrt{A_c^2 + A_s^2},$$
 (5)

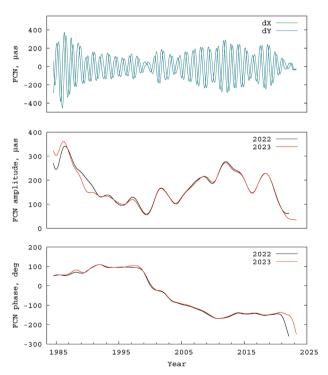
Thus obtained the FCN amplitude and phase variations are shown in Figure 4. For a better representation of the FCN phase variations, the linear trend corresponding to the FCN frequency was removed from the FCN phase se-

② https://ivscc.gsfc.nasa.gov/products-data/product-tables/bkg-products-eops.html

ries. The results of the previous analysis [18] are also shown in these plots. Note that the IVS EOP series [18] ended in February 2022 (epoch 2022.13).

Results presented in Figure 4 confirmed the deep minimum of the FCN amplitude and FCN phase jump discussed <sup>[18]</sup>, but their epoch is later than was suggested <sup>[18]</sup> by about one year. This epoch still cannot be accurately determined because the minimum of the FCN amplitude most probably is not reached yet. However, it can be noted that the current minimum of the FCN amplitude, which is  $\approx$  35 mas, is deeper than the previous minimum in about 1999.4, which was  $\approx$  55 mas.

One can also see in Figure 4 that the amplitude variations in the 1980s and at the beginning of the 1990s are substantially different for the previous and the current analysis. This can be explained by using different versions of the IVS CPO series [18] and in this study. These series substantially differ at the beginning of the interval due to the low accuracy and instability of the CPO observations in this period. The most reliable CPO data begins in May 1993 [19].



**Figure 4**. Free core nutation (FCN) series (upper panel), FCN amplitude (middle panel), and FCN phase minus linear drift (bottom panel). In the amplitude and phase plots, data from the previous publication are shown in black, and data obtained in this work are shown in red.

#### 4. Conclusions

In this paper, a re-computation was made of the CW

and FCN amplitude and phase variations using the latest data of EOP observations. Obtained results were compared with the results of the prediction of the recent peculiarities in the CW and FCN behavior published earlier. The comparison confirmed the deep minima of the CW and FCN amplitude and simultaneous CW and FCN phase jumps, but the epochs of these events occurred to be later than predicted. Such a result could be expected because the recent amplitude and phase variations under investigation are detected near the end of the EOP series used for the study. Therefore, the addition of new data obtained during the last years allowed us to determine this epoch more accurately. Also, an edge effect in the data analysis can impact the accuracy of the obtained epochs of the minima of the CW and FCN amplitude and phase jumps.

It should be also noted that the phase of any physical oscillation is often less stable during the period of near-zero amplitude. This effect is also observed in both the case of CW and FCN.

It is important to bear in mind that the parts of the phase variation plots in Figures 2 and 4 close to horizontal do not mean that the phase of corresponding oscillation (CW or FCN) is constant, since the slope of the line on the plot directly depends on the linear drift corresponding to the CW (FCN) frequency which has been subtracted from the phase before plotting. Therefore, these periods simply correspond to the periods of linear phase change. However, the jumps and other disturbances in the plots correspond to the actual non-linear disturbances in the CW or FCN phase.

From results of the analysis of the CW and FCN series showed that in both cases the precise epochs of the current minima of the CW (to a lesser extent) and FCN (to a larger extent) amplitude and simultaneous phase jumps are still hardly possible. Re-computation of the CW and FCN series in two-three years with the addition of new observations will allow us to investigate these events in more detail.

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<sup>3</sup> https://ui.adsabs.harvard.edu/

<sup>4</sup> http://www.gnuplot.info/

#### **Data Availability Statement**

IERS EOP series are available at https://datacenter.iers.org/eop.php. Author's FCN and CPO series used in this study are available at http://www.gaoran.ru/english/as/persac/. Other results derived from this work can be available from the author upon reasonable request.

#### **Conflicts of Interest**

The author states that there are no known conflicts of interest.

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