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**ARTICLE**

# Partition Optimization of Hydroelectricity Power System and Appropriate Option of Renewable Energy Source in Terms of Probabilistic Multi-Objective Optimization

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## ABSTRACT

The partition optimization and option of renewable energy source for specific place are basic problems which include multiple objectives, such as cost, benefit, and adjustable performance, etc. Particularly, partition optimization is a specific optimal design under the constraint condition of the summation of the proportion of each component being 100%, i.e., a "mixture design" problem in principle. In this paper, the combination of probabilistic multi-objective optimization (PMOO) with uniform design for mixture (UDM) is employed to solve the problems of partition optimization and the option of renewable energy source for specific place. In the study, PMOO is used to converse the multi-objective optimization problem (MOO) into a mono-objective one, and UDM with discretization treatment is used to provide a greatly simplified assessment with a set of homogeneous sampling points in the optimization design with the constraint condition of the summation of total partition ratios being 100% specifically. In the optimization of partition ratios of a hydroelectricity power system, the total estimated expenditure is minimized, and the annual average power generation of three hydropower stations is maximized in the system. It gives the rounding-off optimum partitions of the three hydropower stations as 66 kW, 55 kW and 109 kW under the condition of a total installed capacity of 230 kW, respectively; the total cost and annual power generation are 4.3251 billion yuan and 127.7356 billion degrees correspondingly. Subsequently, the study on the selection of renewable energy source in specific place in India results in solar energy as the appropriate option.

***Keywords:*** Partition; PMOO; Preferable Probability; Uniform Design for Mixture; Discretization

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# 1. Introduction

Distribution of installed capacity of hydroelectricity power stations is a typical optimization problem with multiple objectives (MOO) in the optimal option. In a hydropower station system, not only is the investment of a system concerned, but also the power generation, the adjustable performance (power generation quality), the comprehensive utilization benefit of hydropower resources in a system, etc., are all involved, and the benefits of individual power stations need to be considered as well [1]. Therefore, the proper distribution of installed capacity is undoubtedly a multi-objective optimization problem.

On the other hand, the increase in populations and industrialization aggravates the energy consuming in the world; it experiences a tremendous rise in energy demand due to growing economies and modernization [2]. As a result, some countries or regions are even faced with an energy crisis due to the contradiction between their demands and the supply of fossil energy resources [3,4]. Therefore, the exploration of renewable energy (RE) has been increasing rapidly worldwide. Globally, there are various renewable energy resources available in different regions; therefore, an appropriate selection and utilization of these renewable energy sources are quite important for an explorer to achieve efficient and reasonable application. Unavoidably, the appropriate option of these renewable energy sources involves the comprehensive assessment and comparison of each RE resource in financial, technical, infrastructure management and maintenance, land requirements, and socio-economic aspects, etc. [4]. Therefore, it is a simultaneous optimization problem with multiple objectives. These multiple objectives also involve uncertainty and conflicting criteria. Inevitably, researchers have to employ proper approaches to multi-objective optimization (MOO) to address the matter.

Commonly, in traditional approaches (in fact, algorithms solely) of MOO, it is difficult to find an optimal solution for the MOO problem; it could only yield a so-called non-inferior solution or satisfactory solution instead of an actual optimal solution as a whole, such as those of the Pareto front, etc. [5,6].

Although there have been many algorithms to find satisfactory solutions, they are all problematic. Roughly, the algorithms can be divided into two categories. One is hierarchical analysis based on vector optimization theory and utility theory. The non-inferior solution set is obtained in a certain way first, and then the utility function is constructed or the scheme is optimized by weighted average. The other algorithm is a multi-stage, multi-level and even multi-objective fuzzy optimization approach. It selects the satisfactory solution from the non-inferior solution set according to the minimum Euclidean weighted distance. Both of the above algorithms are based on the comparison and screening of a limited number of schemes in the non-inferior solution set of intermediate problems, and the premise is to form a non-inferior solution set containing a limited number of schemes first. In addition, it contains unknown parameters, such as weight factors, normalized factors, "virtual solution", etc.

A more serious problem is that it adopts the operation mode of "additive algorithm", which actually has the meaning of "union" from the perspective of set theory. However, the essence of MOO is to conduct the optimization of multiple objectives at the same time, which should belong to the mode of "intersection" from the perspective of set theory. Therefore, the actual case is that the past algorithms could not reflect the scientific connotation of the simultaneity of MOO [5–8]. A Pareto solution, also known as a non-inferior solution or an efficient solution, is a concept of a solution in multi-objective optimization. Pareto optimality was proposed by the Italian economist Pareto in 1879 and thereafter. Pareto solutions are generally not unique. The key to solving such multi-objective optimization problems lies in selecting the most satisfactory result according to some rational criterion. Subsequently, numerous multi-objective optimization algorithms have been developed to obtain the Pareto front instead of the actual optimal solution [9–18]. What kind of results can these algorithms provide? The analysis shows that the results they can provide include: 1. Pareto optimal set: a collection of non-dominated solutions, each representing a trade-off; 2. Pareto front: a visualization of these solutions' performance in the objective space, forming a curve (or surface). For example: (1) In a portfolio optimization, the Pareto front may illustrate the trade-off between "risk vs. return"; (2) In engineering design, it may show the trade-off between "strength vs. weight". The practical significance of these algorithms is that they offer "all possible trade-offs" for decision-makers to choose from actual needs instead of a "single correct answer". Although these methods have been applied in many fields to varying degrees, they have inherent defects.

This situation indicates the necessity of an appropriate approach that could reveal the simultaneity of optimization with multiple objectives actually. Improper selection and utilization of approaches for MOO often result in invaluable or misleading consequences.

Recently, probabilistic multi-objective optimization (PMOO) was proposed to deal with the optimization problem with multiple objectives concurrently in a system [5,6], which aims to reveal the intrinsic simultaneity of the multiple objectives in optimization within the system. Besides, the optimization of a system being the whole / integral optimization of the system is taken into consideration as the fundamental view.

The fundamental thought of PMOO is based on the following considerations [5,6]. Multi-objective optimization refers to the situation where the object (system) under study contains multiple objectives that cannot be separated and must be optimized simultaneously. This is its completeness. For example, a material used in aircraft manufacturing needs to be strong and durable, as well as lightweight and resistant to environments. Many material properties (objectives) are involved, such as strength, toughness, lightweight, and environmental resistance, which are inseparable indicators within the material and cannot be isolated. In other words, certain constraints must be considered to achieve high strength and lightweight simultaneously in a material, because high strength and lightweight (low density) are conflicting indicators which coexist within the material; they are mutually restricting each other and being inseparable.

Indeed, in multi-objective optimization problems, these properties need to be optimized simultaneously to enable the system as a whole to function/work properly! These properties are interrelated and constrained, so it is impossible to adjust one without affecting the others! The system's environmental adaptability is also a crucial performance metric for its reliability and success. Therefore, from a systems theory perspective, a multi-objective optimization problem is essentially an optimization of the entire system. Each objective is an organic part of the system.

Addressing the issue of optimizing multiple objectives simultaneously involves treating the system as a whole for optimization, i.e., putting the entire system in the best possible state, with its various parts working together and uniting under the banner of "overall optimization".

Only when all parts (objectives) of the system are coordinated can the system reach an integrated optimization and realize the overall function of the system. Therefore, according to systems theory, the optimization of the system should be: (1) overall optimization; (2) hierarchical optimization at each stage; (3) collaboration among all parties.

Additionally, from the perspective of systems theory, "the whole system does not equal the simple sum of its components" but rather "the whole is greater than the sum of its parts".

However, previous (so-called) multi-objective optimization algorithms, such as linear weighting or "ε-constraint", which treat "weighted summation" or "selecting one of the k objectives" as the optimization goal while converting the remaining (k-1) objectives into constraints, they all diverge from the essence of "simultaneous optimization of multiple objectives" and deviate from the intrinsic nature of "overall system optimization". Moreover, the selection of weight factors and normalization factors for each attribute in the "linear weighting" method also poses issues.

Given that the essence of "multi-objective optimization" means "simultaneous optimization of multiple objectives", it is fundamentally "overall system optimization". Therefore, it is necessary to find the "intersection" between objectives, making them coordinate with each other to achieve optimal functionality of the entire system.

The concept of "intersection" comes from set theory, involving two sets A and B, where the intersection is the set consisting of all elements that belong to both A and B, denoted as A·B or A∩B. In probability theory, the probability P(A∩B) of two independent events occurring simultaneously equals P(A)·P(B), i.e., P(A∩B) = P(A)·P(B), known as the joint probability of the two independent events A and B. Additionally, from the perspective of systems theory, systems can take various forms, and each component within a system can also have diverse forms. Materials in multi-objective material selection have the form of an entity system, while their properties such as elastic modulus, tensile strength, and elongation more closely resemble conceptual systems. For problems involving simultaneous optimization of multiple objectives, Derringer et al. and Jorge et al. transform each objective into its satisfaction value and then combine all satisfaction values using their geometric mean to obtain a total satisfaction value representing the overall evaluation of this combination [19,20]. However, from a probabilistic standpoint, this patchwork approach fundamentally does not align with the essence of optimizing multiple objectives simultaneously [5,6]. Therefore, it is necessary to construct a methodological framework for simultaneous optimization of multiple objectives from the perspectives of systems theory and probability theory rationally, which leads to the emergence of probability-based multi-objective optimization methods.

In probability theory, the "product" of the probabilities of two individual events reflects the "simultaneous occurrence" of these events, and the "intersection" of two individual subsets reveals the "simultaneous occurrence" in both subsets in set theory. Therefore, if the probability theory method is adopted to conduct the issue of simultaneous optimization of multiple objectives, the subsequent problem is to quantify the attribute response of all objectives (attributes) of alternative candidates. So, in PMOO, the preference degree of an attribute in the optimization was reflected by a novel idea called "preferable probability" [2,3]. Moreover, all objectives (attributes) of candidate schemes in the optimization are grouped into either beneficial kinds of attributes or unbeneficial kinds of attributes preliminarily. Furthermore, a quantitative evaluation of each attribute to its corresponding candidate scheme is performed by a new index called partial preferable probability initially [5,6].

Thus, the product of all its partial preferable probabilities of a candidate scheme leads to a total preferable probability, which is the unique index of this candidate scheme. Finally, the total preferable probability of each scheme is its uniquely decisive representative to compete in the optimization process undoubtedly [5,6].

Obviously, PMOO is promised to perform the concurrent optimization problem of multiple objectives as a whole, so as to get effective and valuable consequences. The assessments of PMOO are introduced as follows briefly.

For beneficial kinds of objectives [5,6], the partial preferable probability $P_{ij}$ is evaluated with Equation (1),

$$\begin{aligned} &P_{ij} = \gamma_j \chi_{ij} \\ &\gamma_j = 1/(n\overline{\chi_j}) \\ &i = 1,2,\cdots,n \\ &j = 1,2,\cdots,m \end{aligned} \tag{1}$$

As to unbeneficial types of objectives [5,6], it has,

$$\begin{aligned} &P_{ij} = \eta_j(\chi_{j\min} + \chi_{j\max} - \chi_{ij}) \\ &\eta_j = 1/[n(\chi_{j\min} + \chi_{j\max} - \overline{\chi_j})] \end{aligned} \tag{2}$$

Finally, the total preferable probability $P_i$ of the $i$-th alternative scheme is [5,6],

$$P_i = P_{i1} \cdot P_{i2} \cdot P_{i3} \cdots P_{im} = \prod_{j=1}^{m} P_{ij} \tag{3}$$

In Equation (1) through Equation (3), $\chi_{ij}$ is the performance utility index of the $j$-th objective of the $i$-th candidate; n is the total number of alternatives; m reflects the total number of objective; $\overline{\chi_j}$ indicates the mean value of the utility index of the $j$-th objective within the involved candidates; $\chi_{jmin}$ and $\chi_{jmax}$ are the minimum and maximum values of the performance utility index involved in $j$-th objectives, respectively; $\gamma_j$ and $\eta_j$ indicate the normalization factors of the $j$-th utility index $\chi_{ij}$ in cases of beneficial kinds and unbeneficial kinds, respectively [2,3]. In addition, the probabilistic robust design of the production process and product was developed on the basis of the viewpoint of systems theory [21,22].

In fact, there exist distinct differences between PMOO and traditional approaches for MOO problems. The main differences are reflected in the fact that PMOO contains both the viewpoint of systems theory and mathematical algorithms. From the point of view of systems theory, "the optimum point of the optimization problem of multiple objectives" is the "optimal point of the system", and the latter can be obtained by using probability theory. However, traditional approaches for MOO problems lack any viewpoint for the optimum point but only algorithms, which lead to the consequence that the optimum point of the "optimization problem with multiple objectives" is not defined.

Besides, partition optimization is a specific optimal design problem, which has a typical characteristic of the ratios of input partition variables usually being restrained by common constraint conditions, $x_{i0} \geq 0$, $i$ = 1, 2, ..., $s$, and $\sum_{i0=1}^{s} x_{i0} = 1$ [23], in which $x_{i0}$ reflects the $i$-th input partition variable and $s$ indicates the number of input partition variables. Such kind of problem with the above constraints for input variables is called "mixture design" in the chemical industry, material manufacturing industry and design of food formulas [23]. Therefore, the uniform design for the mixture problem (UDM) proposed by Fang et al. could be employed to deal with the problem of partition optimization [23]; it provides a set of homogeneous sampling points in the mixture design under the above constraint conditions. In fact, the uniformly distributed sampling points can be used to conduct the subsequent data processing in the optimization process by means of discretization especially.

In this paper, the combination of PMOO with UDM is used to solve the problem of partition optimization of hydroelectricity power system in order to get an appropriate option first. The total calculation expenditure of a hydroelectricity power system and its annual average power generation are taken as the dual objectives to get global optimization with proper partition distributions of installed capacity of three hydropower stations. Subsequently, appropriate selection of renewable energy source for specific place is conducted in terms of probabilistic multi-objective optimization.

# 2. Materials and Methods

## 2.1. Uniform Design for Mixture Design with Three Input Variables

Uniform Design for Mixture Design (UDM) was proposed by Fang et al. [21], it provides a homogeneous sampling points in mixture design under constraint conditions of $x_{i0} \geq 0$, $i$ = 1, 2, ..., $s$, and $\sum_{i0=1}^{s} x_{i0} = 1$ [23]. In detail, Fang et al. established a procedure to deduce a specific table $UM_r(r^s)$ to spread the sampling points in the input variable – space accordingly [23].

As to a UDM with three input variable ratios ($s$ = 3), i.e., $x_{10}$, $x_{20}$, $x_{30}$, the number of sampling points $r$ = 19 can be employed in our treatment [5]; the corresponding uniform design table $U^*_r(r^t)$ or $U_r(r^t)$ and its usage table can be chosen accordingly [23,24]. Under such conditions, the number of columns of the usage table could be set as $s$ – 1.

While, the elements in the original uniform design table $U^*_r(r^t)$ or $U_r(r^t)$ are marked by $\{q_{ik}\}$ [23,24]. Accordingly, the following procedures are formulated [23,24].

(I) Construction of a novel element $c_{ki}$

As to each $i$, a novel element $c_{ki}$ is built by using the following formula [23,24],

$$c_{ki} = (2q_{ki} - 1)/(2r) \tag{4}$$

(II) Construction of uniform sampling points for the mixtures, $y_{ki0}$

Following formula is employed to complete the construction [23,24],

$$x_{ki0} = \left(1 - c_{ki}^{\frac{1}{s-j}}\right) \prod_{j=1}^{i-1} c_{ki}^{\frac{1}{s-j}}, i = 1,2,\ldots,(s-1) \tag{5}$$

$$x_{ks0} = \prod_{j=1}^{s-1} c_{kj}^{\frac{1}{s-j}}, k = 1,2,\ldots,r \tag{6}$$

Thus, $\{x_{ki0}\}$ is deduced, which could be employed to build the corresponding uniform design table $UM_r(r^s)$ of the mixture under the conditions of specific $n$ and $s$.

Based on the uniform design table $U^*_{19}(19^7)$ of Fang et al. [23,24], a uniform test table $UM_{19}(19^3)$ with mixtures can be built, which is shown in **Table 1**. Besides, here we have $s$ = 3 for three input variables, $r$ = 19 for the number of sampling points, so it derives the following expressions from the above rules,

$$\begin{aligned} x_{k10} &= 1 - c_{k1}^{0.5} \\ x_{k20} &= c_{k1}^{0.5} \cdot \left(1 - c_{k2}\right) \\ x_{k30} &= c_{k1}^{0.5} \cdot c_{k2} \end{aligned} \tag{7}$$

**Table 1.** Uniform test table $UM_{19}(19^3)$ with mixtures based on uniform design table $U^*_{19}(19^7)$.

| No. | $q_{10}$ | $q_{20}$ | $c_1$ | $c_2$ | $x_{10}$ | $x_{20}$ | $x_{30}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 9 | 0.0263 | 0.4474 | 0.8378 | 0.0896 | 0.0726 |
| 2 | 2 | 18 | 0.0789 | 0.9211 | 0.7190 | 0.0222 | 0.2588 |
| 3 | 3 | 7 | 0.1316 | 0.3421 | 0.6373 | 0.2386 | 0.1241 |
| 4 | 4 | 16 | 0.1842 | 0.8158 | 0.5708 | 0.0791 | 0.3501 |
| 5 | 5 | 5 | 0.2368 | 0.2368 | 0.5133 | 0.3714 | 0.1153 |
| 6 | 6 | 14 | 0.2895 | 0.7105 | 0.4620 | 0.1557 | 0.3823 |
| 7 | 7 | 3 | 0.3421 | 0.1316 | 0.4151 | 0.5079 | 0.0770 |
| 8 | 8 | 12 | 0.3947 | 0.6053 | 0.3717 | 0.2480 | 0.3803 |
| 9 | 9 | 1 | 0.4474 | 0.0263 | 0.3311 | 0.6513 | 0.0176 |
| 10 | 10 | 10 | 0.5000 | 0.5000 | 0.2929 | 0.3536 | 0.3536 |
| 11 | 11 | 19 | 0.5526 | 0.9737 | 0.2566 | 0.0196 | 0.7238 |
| 12 | 12 | 8 | 0.6053 | 0.3947 | 0.2220 | 0.4709 | 0.3071 |
| 13 | 13 | 17 | 0.6579 | 0.8684 | 0.1889 | 0.1067 | 0.7044 |
| 14 | 14 | 6 | 0.7105 | 0.2895 | 0.1571 | 0.5989 | 0.2440 |
| 15 | 15 | 15 | 0.7636 | 0.7632 | 0.1264 | 0.2069 | 0.6667 |
| 16 | 16 | 4 | 0.8158 | 0.1842 | 0.0968 | 0.7368 | 0.1664 |
| 17 | 17 | 13 | 0.8684 | 0.6579 | 0.0681 | 0.3188 | 0.6131 |
| 18 | 18 | 2 | 0.9211 | 0.0789 | 0.0403 | 0.8839 | 0.0758 |
| 19 | 19 | 11 | 0.9737 | 0.5526 | 0.0132 | 0.4414 | 0.5453 |

## 2.2. Combination of PMOO with UDM

In light of uniform design for mixture design (UDM), the assessments of continuous functions of objectives within their valid domain can be replaced by the evaluations of the limited number of values of the corresponding objectives at the typical sampling points, which greatly simplifies assessments.

In order to perform assessments of PMOO, the values of objective functions at every sampling point can be conducted first accordingly. Subsequently, the corresponding result of partial preferable probability for each objective at every sampling point, and the total preferable probability at every sampling point can be conducted.

All these procedures form the Combination of PMOO with UDM, which makes it possible for PMOO evaluations at the typical sampling points of UDM.

The sampling points provided in **Table 1** are for the case of partition optimization with three input partition variables only, while for other numbers of input variables it can be conducted according to Fang et al. [23,24]. Two application examples are provided in this section.

# 3. Results

## 3.1. Application of Combination of PMOO with UDM in Partition Optimization of Hydroelectricity Power System

In the optimization problem of distributions of installed capacity of the hydropower system, it often involves the total investment, the system power generation, power generation quality (adjustable performance), and the comprehensive utilization benefit of hydropower resources, etc. In this section, the allocation of installed capacity of hydropower stations in a system is optimized with the total expenditure and the average power generation for many years as dual objectives of the system.

Li, Shang and Huang raised a partition problem of three hydropower stations in a system power generation [1], which involves minimizing total estimated expenditure $C$, of the system and maximizing the annual average power generation $E$ in China. **Table 2** shows the relationship among the installed capacity $y$ of three hydropower stations, the estimated expenditure $C$ and annual average power generation $E$ in the power system from Li, Shang and Huang [1].

**Table 2.** Data of relationship among the installed capacity of 3 hydropower stations, estimated expenditure and average annual power generation [1].

| No. | Installed Capacity $y$ (kW) | Cost $C$ (billion yuan) | Annual Power Generation $E$ (billion degree) |
|---|---|---|---|
| Station 1 | $y_1$ | $C_1$ | $E_1$ |
| | 60 | 1.5780 | 32.0 |
| | 70 | 1.5890 | 36.5 |
| | 80 | 1.6050 | 41.1 |
| | 90 | 1.6117 | 43.6 |
| | 100 | 1.6335 | 45.5 |

| No. | Installed Capacity $y$ (kW) | Cost $C$ (billion yuan) | Annual Power Generation $E$ (billion degree) |
|---|---|---|---|
| Station 2 | $y_2$ | $C_2$ | $E_2$ |
| | 40 | 1.5200 | 27.0 |
| | 50 | 1.5460 | 32.6 |
| | 60 | 1.5680 | 37.7 |
| | 70 | 1.5910 | 42.1 |
| | 80 | 1.6200 | 45.4 |
| Station 3 | $y_3$ | $C_3$ | $E_3$ |
| | 80 | 1.5800 | 41.9 |
| | 90 | 1.6120 | 48.5 |
| | 100 | 1.6520 | 53.0 |
| | 110 | 1.6900 | 57.09 |
| | 120 | 1.7280 | 61.0 |

The restraint condition of the hydropower system is that the total installed capacity of the hydropower stations is 230 kW in the system [1], i.e., $y_1 + y_2 + y_3 = 230$ kW, and the ranges of installed capacity of each power station were $y_1 \in [60, 100]$ in kW, $y_2 \in [40, 80]$ in kW and $y_3 \in [80, 120]$ in kW, respectively [1]; here $y_i$ indicates the installed capacity of the $i$-th each power station, $i = 1, 2, 3$.

Furthermore, it is stated that this optimization problem is to take the minimizing total estimated expenditure $C$ of the system and maximizing the annual average power generation $E$ as dual objectives under the restraint conditions of total installed capacity of the three hydropower stations in the system being 230 kW and the corresponding ranges of installed capacity of each power station.

From the data in **Table 2**, the following regressed relationships can be obtained,

$$C_1 = 9\times10^{-6} y_1^2 - 7\times10^{-5} y_1 + 1.551, R^2 = 0.9827 \tag{8}$$

$$E_1 = -0.0052 y_1^2 + 1.1753 y_1 - 19.869, R^2 = 0.998 \tag{9}$$

$$C_2 = 5\times10^{-6} y_2^2 + 0.0019 y_2 + 1.439, R^2 = 0.9983 \tag{10}$$

$$E_2 = -0.0038 y_2^2 + 0.9173 y_2 - 3.691, R^2 = 0.9998 \tag{11}$$

$$C_3 = 7\times10^{-6} y_3^2 + 0.0023 y_3 + 1.348, R^2 = 0.9994 \tag{12}$$

$$E_3 = -0.0041 y_3^2 + 1.295 y_3 - 35.022, R^2 = 0.998 \tag{13}$$

The optimum problem can be expressed as an optimization issue with dual objectives in the following formulae,

$$\min C = C_1 + C_2 + C_3 \tag{14}$$

$$\max E = E_1 + E_2 + E_3 \tag{15}$$

$$\text{s.t.: } y_1 + y_2 + y_3 = 230 \text{ kW} \tag{16}$$

$$\text{s.t.: } 60kW \le y_1 \le 100kW,\ 40kW \le y_2 \le 80kW,\ 80kW \le y_3 \le 120kW \tag{17}$$

Obviously, this problem is an optimization issue of dual objectives with mixture design [5,23]. Since the objective functions $C$ and $E$ are continuous functions in the independent partition variable space consisting of $y_1$, $y_2$ and $y_3$ now, an effective treatment is to deal with it by using a discretizing algorithm in the independent variable space with a certain number of sampling points [5,23].

Subsequently, 19 uniformly distributed sampling points are necessary to be employed to conduct this mixture design issue in 3-D space fundamentally in light of the study on the number of sampling points [5,21,23], which is shown in **Table 3**.

**Table 3.** 19 uniformly distributed sampling points for this mixture design issue.

| No. | Position of Sampling Point | | |
|---|---|---|---|
| | $y_1$ (kW) | $y_2$ (kW) | $y_3$ (kW) |
| 1 | 97.76162 | 45.56711 | 86.67120 |
| 2 | 92.79553 | 42.74579 | 94.45861 |
| 3 | 89.37638 | 51.79781 | 88.82575 |
| 4 | 86.59717 | 45.12443 | 98.27834 |
| 5 | 84.19402 | 57.34951 | 88.45641 |
| 6 | 82.04610 | 48.33113 | 99.62271 |
| 7 | 80.08608 | 63.05918 | 86.85468 |
| 8 | 78.27187 | 52.18931 | 99.53876 |
| 9 | 76.57509 | 69.05245 | 84.37240 |
| 10 | 74.97551 | 56.60313 | 98.42130 |
| 11 | 73.45814 | 42.63626 | 113.90550 |
| 12 | 72.01145 | 61.50979 | 96.47870 |
| 13 | 70.62641 | 46.28120 | 113.09230 |
| 14 | 69.29575 | 66.86399 | 93.84019 |
| 15 | 68.01353 | 50.47046 | 111.51590 |
| 16 | 66.77481 | 72.63104 | 90.59409 |
| 17 | 65.57544 | 55.15001 | 109.27450 |
| 18 | 64.41191 | 78.78325 | 86.80478 |
| 19 | 63.28116 | 60.27850 | 106.44030 |

Moreover, the values of functions $C$ and $E$ at these

19 sampling points are evaluated, which are shown in **Table 4**.

**Table 4.** Values of functions $C$ and $E$ at the 19 sampling points.

| No. | Value of Function | |
|---|---|---|
| | **$C$ (billion yuan)** | **$E$ (billion degree)** |
| 1 | 4.367772 | 121.9677 |
| 2 | 4.344959 | 123.7122 |
| 3 | 4.366799 | 124.9223 |
| 4 | 4.337318 | 125.5252 |
| 5 | 4.373035 | 126.0890 |
| 6 | 4.337090 | 126.6202 |
| 7 | 4.383253 | 126.4722 |
| 8 | 4.341254 | 127.3559 |
| 9 | 4.396629 | 126.2229 |
| 10 | 4.348747 | 127.7925 |
| 11 | 4.300760 | 126.2080 |
| 12 | 4.359072 | 127.9096 |
| 13 | 4.306410 | 126.8172 |
| 14 | 4.371971 | 127.6549 |
| 15 | 4.314466 | 127.3428 |
| 16 | 4.387316 | 126.9598 |
| 17 | 4.324759 | 127.7117 |
| 18 | 4.405048 | 125.7471 |
| 19 | 4.337202 | 127.8438 |

The assessments of partial preferable probabilities of functions $C$ and $E$, and the total preferable probabilities at 19 sampling points, as well as ranking, are shown in **Table 5**. In the assessments, the regressed relationships are employed.

**Table 5** shows that the highest total preferable probability appears at the 17th sampling point, which can be selected as the optimized status. The corresponding partition variables $y_1$*, $y_2$* and $y_3$*, are 65.57544 kW, 55.15001 kW and 109.27450 kW, respectively. The total cost $C$* and annual power generation $E$* are 4.324759 billion yuan and 127.7117 billion degrees correspondingly. The rounding-off values are, $y_1'$ = 66 kW, $y_2'$ = 55 kW and $y_3'$ = 109 kW, respectively; the corresponding total cost $C'$ and annual power generation $E'$, are 4.3251 billion yuan and 127.7356 billion degrees individually.

These results are superior to those given by Li, Shang and Huang with their fuzzy multi-objective optimization approach [1], i.e., $y_1''$, $y_2''$ and $y_3''$, are 80 kW, 60 kW and 90 kW, respectively. Their total cost $C''$ and annual power generation $E''$, are 4.785 billion yuan and 127.3 billion degrees correspondingly. Obviously, the total cost $C''$ of Li, Shang and Huang is greater than $C'$ of our approach, and their annual power generation $E''$ is lower than ours.

**Table 5.** Assessment of partial preferable probabilities of functions $C$ and $E$ and total preferable probabilities at the 19 sampling points, as well as ranking.

| No. | Partial Preferable Probability | | Total Preferable Probability | Rank |
|---|---|---|---|---|
| | $P_C$ | $P_E$ | $P_t \times 10^3$ | |
| 1 | 0.0525 | 0.0508 | 2.6645 | 19 |
| 2 | 0.0527 | 0.0515 | 2.7169 | 18 |
| 3 | 0.0525 | 0.0520 | 2.7297 | 16 |
| 4 | 0.0528 | 0.0523 | 2.7616 | 11 |
| 5 | 0.0524 | 0.0525 | 2.7512 | 14 |
| 6 | 0.0528 | 0.0527 | 2.7859 | 10 |
| 7 | 0.0523 | 0.0527 | 2.7530 | 13 |
| 8 | 0.0528 | 0.0530 | 2.7994 | 8 |
| 9 | 0.0521 | 0.0526 | 2.7390 | 15 |
| 10 | 0.0527 | 0.0532 | 2.8041 | 5 |
| 11 | 0.0533 | 0.0526 | 2.8000 | 6 |
| 12 | 0.0526 | 0.0533 | 2.8000 | 7 |
| 13 | 0.0532 | 0.0528 | 2.8099 | 4 |
| 14 | 0.0524 | 0.0532 | 2.7860 | 9 |
| 15 | 0.0531 | 0.0530 | 2.8164 | 2 |
| 16 | 0.0522 | 0.0529 | 2.7610 | 12 |
| 17 | 0.0530 | 0.0532 | 2.8179 | 1 |
| 18 | 0.0520 | 0.0524 | 2.7233 | 17 |
| 19 | 0.0528 | 0.0532 | 2.8127 | 3 |

## 3.2. Application of Probability-Based Multi-Objectives Optimization in Appropriate Option of Renewable Energy Source

In this section, the example for optimal renewable energy source of Husain et al. is reanalyzed with PMOO. Husain et al. once raised the optimal renewable energy source in India [4]. It involves the comparative option of hydropower, solar energy, wind energy and biomass energy, while the assessed criteria include financial response, technical maturity and efficiency, environmental effect, and social benefit, etc. Furthermore, they gathered data for each kind of RE resource [4], which is cited in **Table 6** here for our restudy. Besides, the types of the objectives ($O_1$ through $O_{10}$) are indicated clearly.

**Table 6.** Gathered data for each kind of RE resource.

| Criterion | Total Installed Cost, $ $kW^{-1}$, $O_1$ | O&M Cost, $ $kW^{-1} y^{-1}$, $O_2$ | LCOE, $ $kWh^{-1}$, $O_3$ | Efficiency, %, $O_4$ | Capacity Factor, %, $O_5$ |
|---|---|---|---|---|---|
| Type of Criterion | Non-Beneficial | Non-Beneficial | Non-Beneficial | Beneficial | Beneficial |
| Solar | 596 | 9.000 | 0.038 | 22.00 | 19 |
| Wind | 1038 | 28.000 | 0.040 | 35.00 | 33 |
| Hydro | 1817 | 45.425 | 0.065 | 76.61 | 57 |
| Biomass | 1154 | 46.160 | 0.057 | 84.33 | 68 |

Continue

| Criterion | GHG Emission, g $CO_2$ $kWh^{-1}$, $O_6$ | Land Requirement, $m^2$ $kW^{-1}$, $O_7$ | Job Creation, Job-years $GWh^{-1}$, $O_8$ | Technical Maturity, 1–5 Score, $O_9$ | Social Acceptance, 1–5 Score, $O_{10}$ |
|---|---|---|---|---|---|
| Type of Criterion | Non-Beneficial | Non-Beneficial | Beneficial | Beneficial | Beneficial |
| Solar | 48 | 12 | 0.870 | 4 | 4.58 |
| Wind | 11 | 250 | 0.170 | 4 | 4.17 |
| Hydro | 24 | 500 | 0.270 | 5 | 3.56 |
| Biomass | 230 | 13 | 0.210 | 3 | 4.00 |

Distinctly, the assessment of this issue can be done easily by using the approach of probability-based multi-objectives optimization (PMOO). The evaluated results with rank are presented in **Table 7**. The ranking is conducted with the total preferable probability of each alternative candidate from higher to lower. It reveals that solar energy in the concerned area is the best selection, which has the highest total preferable probability in the optimal comparison. Undoubtedly, as to the optimal assessment of other places, their appropriate criteria (objectives) and their roles might be of course concerned. Other applications of PMOO show bright prospects as well, such as utilizations in portfolio investment, shortest path problems with multiple objectives, and options of thermofluids, etc. [25–29].

**Table 7.** Evaluated results of this issue.

| Type of RE | Partial Preferable Probability | | | | | |
|---|---|---|---|---|---|---|
| | $P_{01}$ | $P_{02}$ | $P_{03}$ | $P_{04}$ | $P_{05}$ | $P_{06}$ |
| Solar | 0.3600 | 0.5014 | 0.3066 | 0.1009 | 0.1073 | 0.2965 |
| Wind | 0.2724 | 0.2950 | 0.2972 | 0.1606 | 0.1864 | 0.3533 |
| Hydro | 0.1181 | 0.1058 | 0.1792 | 0.3515 | 0.3220 | 0.3333 |
| Biomass | 0.2495 | 0.0978 | 0.2170 | 0.3869 | 0.3842 | 0.0169 |

Continue

| Type of RE | Partial Preferable Probability | | | | Total | Rank |
|---|---|---|---|---|---|---|
| | $P_{07}$ | $P_{08}$ | $P_{09}$ | $P_{010}$ | $P_i$´$10^6$ | |
| Solar | 0.3928 | 0.5724 | 0.2500 | 0.2808 | 2.8063 | 1 |
| Wind | 0.2058 | 0.1118 | 0.2500 | 0.2557 | 0.3718 | 2 |
| Hydro | 0.0094 | 0.1776 | 0.3125 | 0.2183 | 0.0096 | 4 |
| Biomass | 0.3920 | 0.1382 | 0.1875 | 0.2452 | 0.0331 | 3 |

## 4. Discussion

Since the total installed capacity of the hydropower stations in a power system in general is limited and fixed, i.e., the ratio of input partition variable is usually restrained by the common constraint condition of the summation of the proportion of each component being 1 ($\sum_{i0=1}^{s} x_{i0} = 1$, $x_{i0} \geq 0$, $i$ = 1, 2, ..., $s$), which is a typical "mixture design" problem, therefore the combination of probabilistic multi-objective optimization (PMOO) with uniform design for mixture (UDM) is proper to be used to solve problems of such partition optimization undoubtedly. The consequences of the application example confirm the validity of the approach.

## 5. Conclusions

The above study indicates that the combination of probabilistic multi-objective optimization with mixture design can be successfully used to conduct partition optimization problems; the probabilistic multi-objective optimization is used to convert optimization problems with multiple objectives into a mono-objective one, and the discretization is conducted by using uniform design with mixture to provide a set of homogeneous sampling points in optimization design.

The evaluated results for the appropriate option of renewable energy resources reveal that solar energy in the specific area is the best selection. The optimal assessment and selection of other places might concern their actual data of criteria exactly. More exploration of probabilistic multi-objective optimization in broad fields is still open and in need.

## Author Contributions

Conceptualization, M.Z. and J.Y.; methodology, M.Z.; validation, J.Y.; formal analysis, M.Z.; writing—original draft preparation, J.Y.; writing—review and editing, M.Z. All authors have read and agreed to the published version of the manuscript.

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## Conflicts of Interest

The authors declare no conflict of interest.

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